Lublin University of Technology
Departament of Machine Design and Mechatronics

Theory of Machines

Lecture 1

Basic terminology and structural analysis
Linkage mechanisms

Lublin 2017

Łukasz Jedliński

Basic terminology and definitions

The **theory of machines and mechanisms** is a chosen area of knowledge in the field of engineering and technical sciences. There are two basic tasks posed to TMM - analysis and synthesis. In **analysis**, the main problems to be solved are:

- structure examination,
- kinematic analysis,
- dynamic analysis.

On the other hand, among the objectives of **synthesis** one distinguishes:

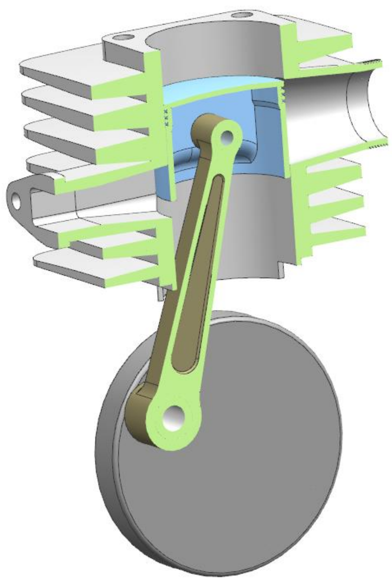
- searching for the structure of mechanisms for given requirements,
- searching for mechanism properties which satisfy the kinematic or dynamic requirements set for them.

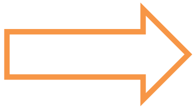
Basic terminology and definitions

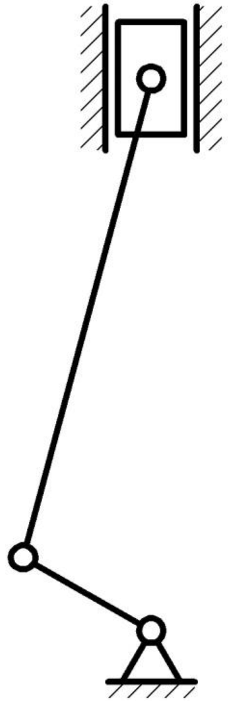
In **analysis**, the main problems to be solved are:

- structure examination,
- kinematic analysis,
- dynamic analysis.

The examination of the structure of machines and mechanisms is aimed at classifying them and determining their mobility. The kinematic diagram made is used in kinematic and dynamic analysis.







Basic terminology and definitions

In **analysis**, the main problems to be solved are:

- structure examination,
- kinematic analysis,
- dynamic analysis.

Kinematic analysis aims to analyze motion by determining the displacement, velocity, and acceleration of the components of a mechanism, neglecting the effects of the mass of the components and the forces applied.

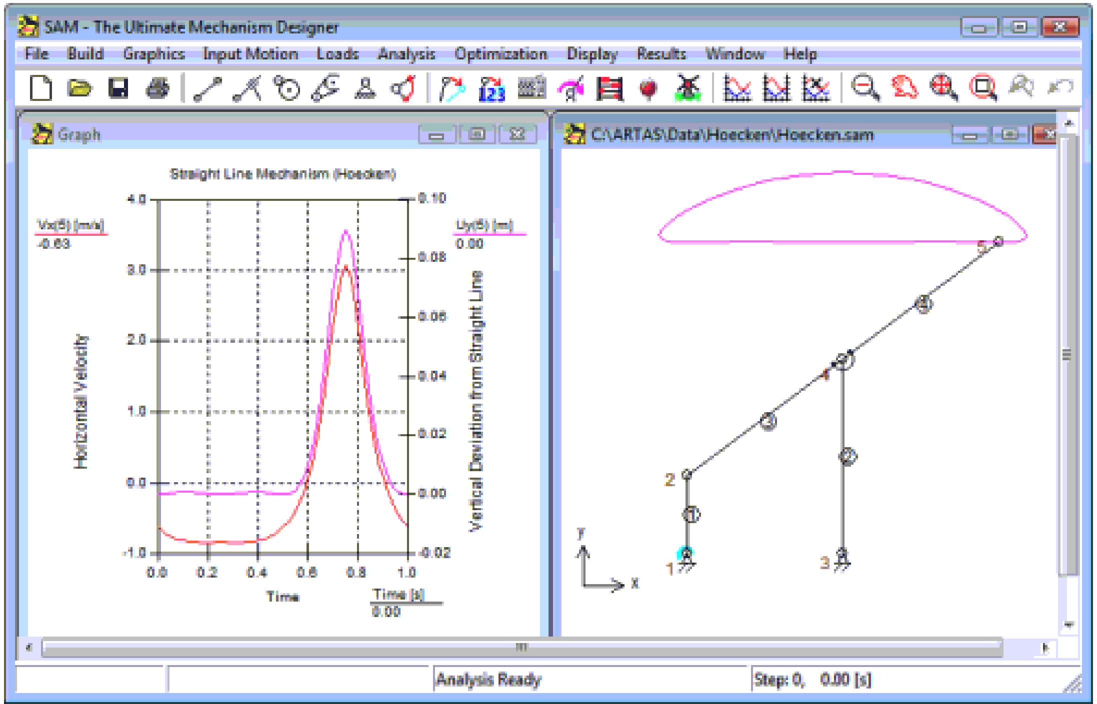


Fig. SAM program (Synthesis and Analysis of Mechanisms) [https://www.artas.nl/en/]

Basic terminology and definitions

In **analysis**, the main problems to be solved are:

- structure examination,
- kinematic analysis,
- dynamic analysis.

Dynamic analysis deals with the study of mechanisms with respect to the forces acting on them. Results from kinematic analysis are used.

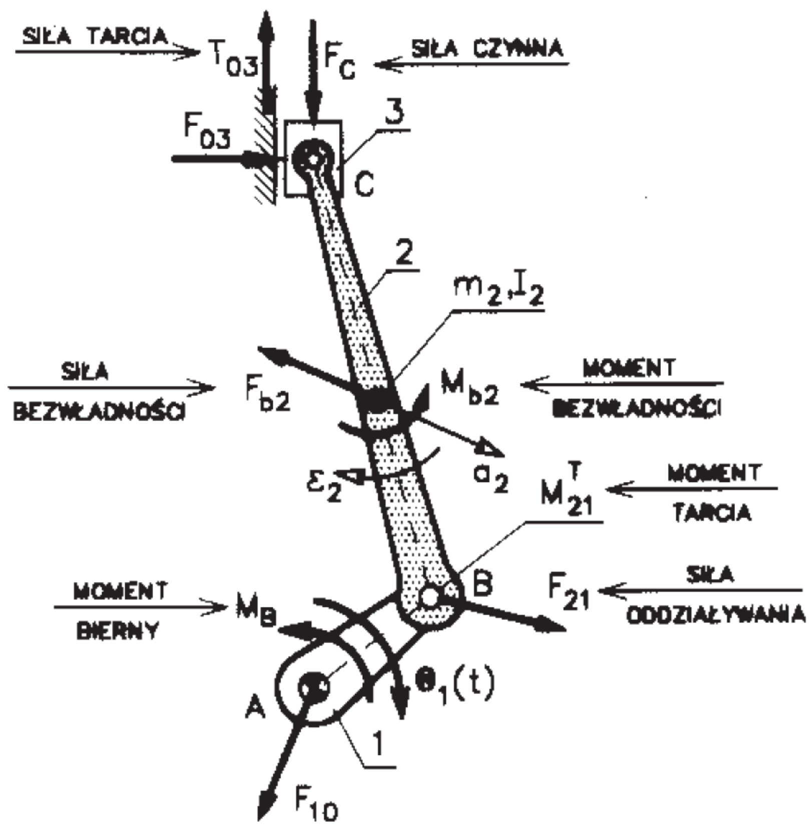


Fig. [Gronowicz 2003]

Basic terminology and definitions

On the other hand, among the objectives of **synthesis** are:

- searching for the structure of mechanisms for given requirements,
- searching for mechanism properties which satisfy the kinematic or dynamic requirements.

Table 1. Possible variants of the kinematic chain [Szrek 2008]

Liczba elementów k	liczba par I klasy p_1	liczba par II klasy p_2	symbol U_i $(k\ p_1\ p_2)$
0	0	1	0 0 1
1	2	0	1 2 0
2	3	1	2 3 1
3	5	0	3 5 0

Table 2. Basic structural diagrams [Szrek 2008]

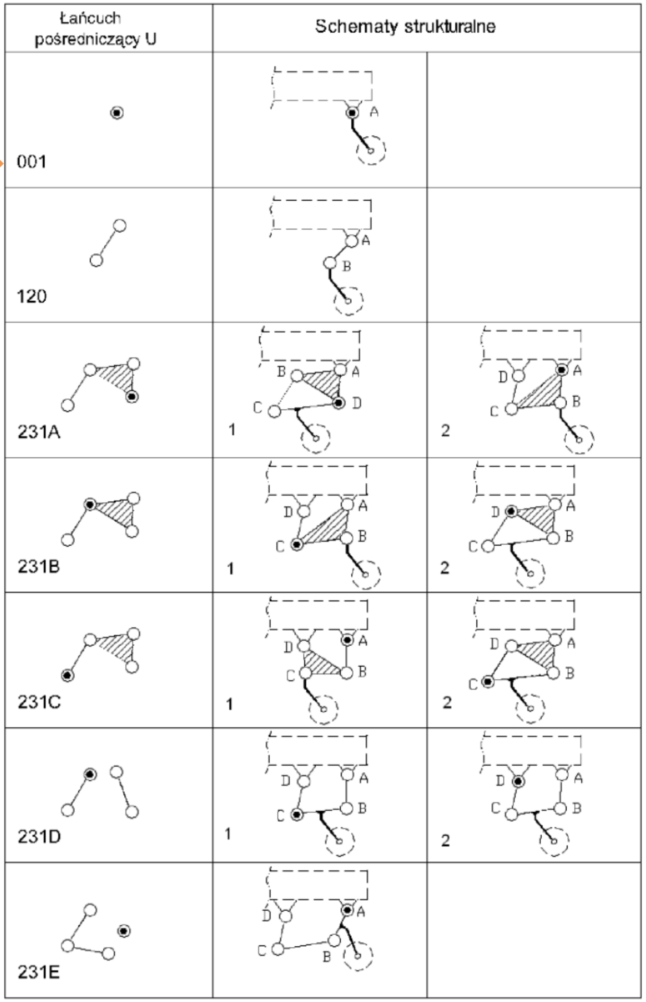
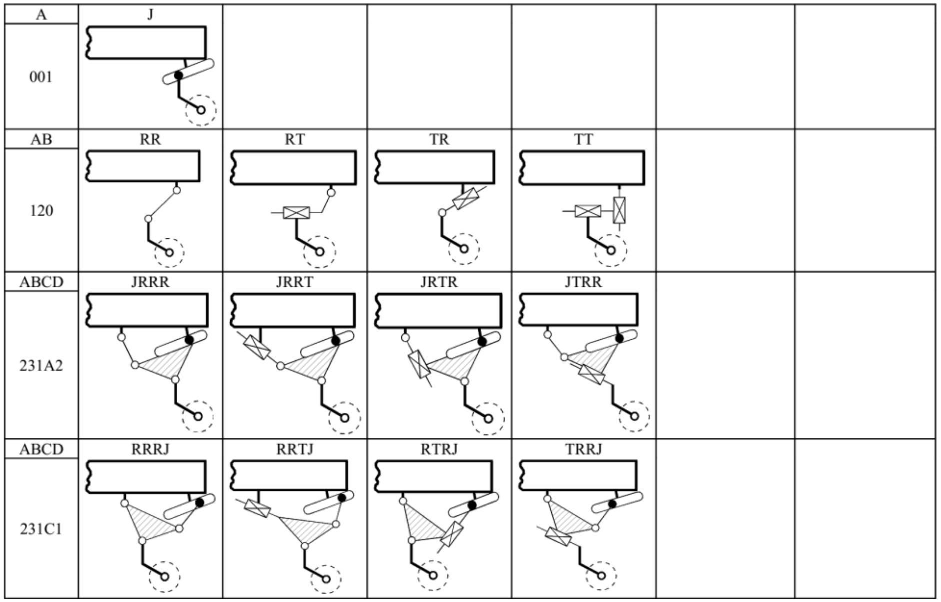


Table 3. Selected kinematic schemes [Szrek 2008]



Basic terminology and definitions

On the other hand, among the objectives of **synthesis** are:

- searching for the structure of mechanisms for given requirements,
- searching for mechanism properties which satisfy the kinematic or dynamic requirements.

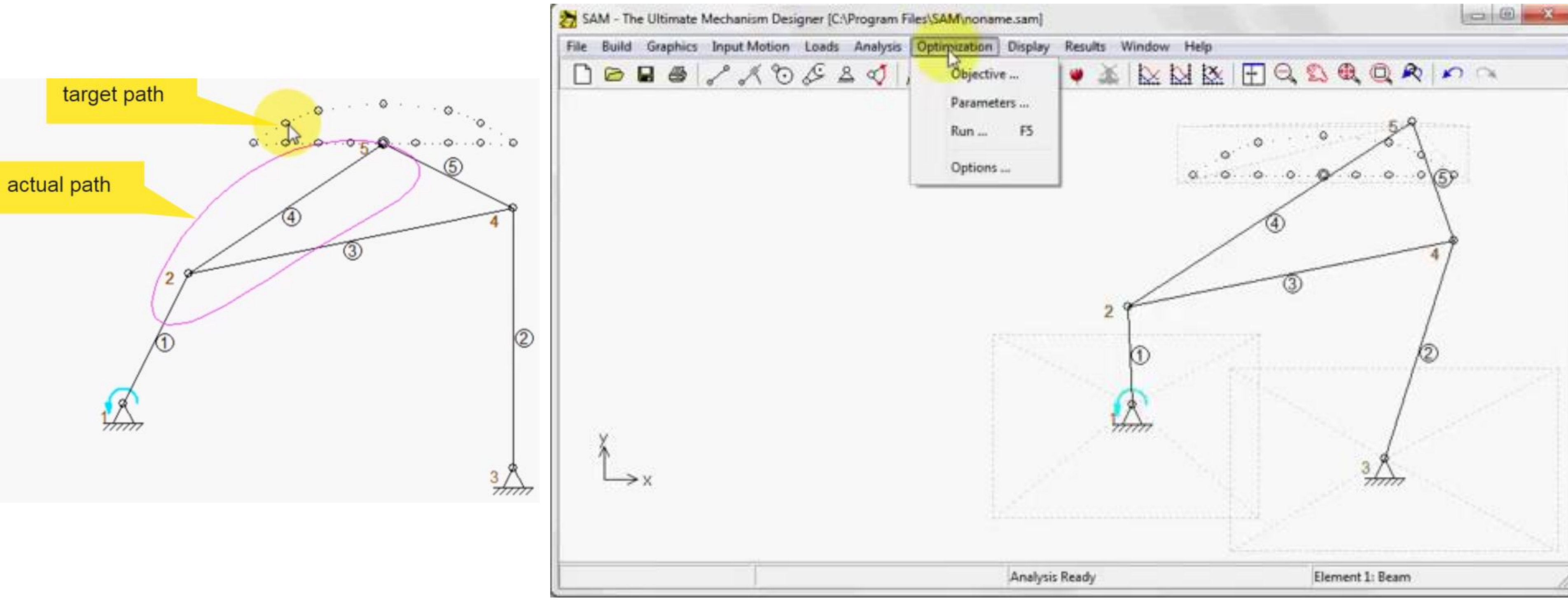


Fig. SAM program [Artas]

Basic terminology and definitions

Mechanism

1. Selected definitions that reject manipulators as mechanisms:

- Franz Reuleaux (1829-1905) – „assemblage of resistant bodies, connected by movable joints, to form a **closed kinematic chain** with one link fixed and having the purpose of transforming motion” [Uicker 2011].

- Miller S. – a closed kinematic chain with one fixed link characterized by the number of driver links equal to its mobility.

A *closed kinematic chain* is a series of links connected by joints such that each of them forms a pair with at least two links.

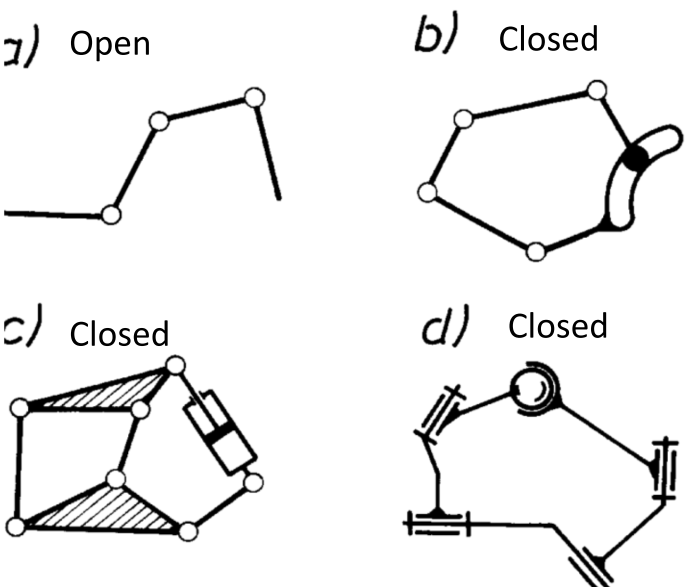


Fig. Kinematic chains [Miller 1989]

Mechanism

2. Selected definitions treating manipulators as mechanisms :

- Felis, Jaworowski, Cieślik – is a kinematic chain performing a precisely defined motion.

A kinematic chain is a set of links connected in kinematic pairs with one fixed.

- Wawrzecki J. – *is a set of connected links with precisely defined relative motion, whose task is to transmit motion* .

There are also definitions that reject natural creations as mechanisms.

Spatial mechanism

Every real mechanism is a spatial mechanism. Planar mechanisms are distinguished from this group for easier kinematic and partly structural and dynamic analysis.

A ***planar mechanism*** is one in which the all points of all members move in planes mutually parallel to some stationary plane.

Basic terminology and definitions

Machine

- Franz Reuleaux (1829-1905) – „combination of resistant bodies so arranged that by their means the mechanical forces of nature can be compelled to do work accompanied by certain determinate motions" [Uicker 2011].

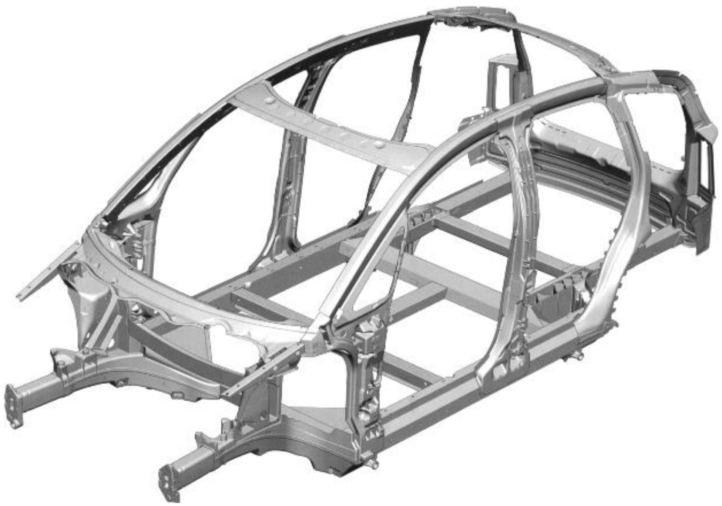
- Felis, Jaworowski, Cieślik – is a set of mechanisms performing to do the desired work related to technological processes or energy transformation.

- Miller S. – is *A device in which a mechanical motion is involved in an power process that results in useful work or energy transformation.*

What is a difference between a machine and a mechanism?

Basic terminology and definitions

Structure – rigid bodies connected in such way that relative motion is not possible and are design to carry load.



Is this an area of interest for TMM?

Fig. Vehicle frame
[https://mechanics.stackexchange.com/tags/frame/info]

Fig. Bridge
[qctimes.com/traffic/photos-new-i--bridge/collection_6fedb8c6-c1ae-11e3-a1b9-0019bb2963f4.html]

Basic terminology and definitions

Link (**member**):
is the element or rigidly connected elements of which a mechanism or machine is made, which mediate in the transmission of motion.

Connecting rod

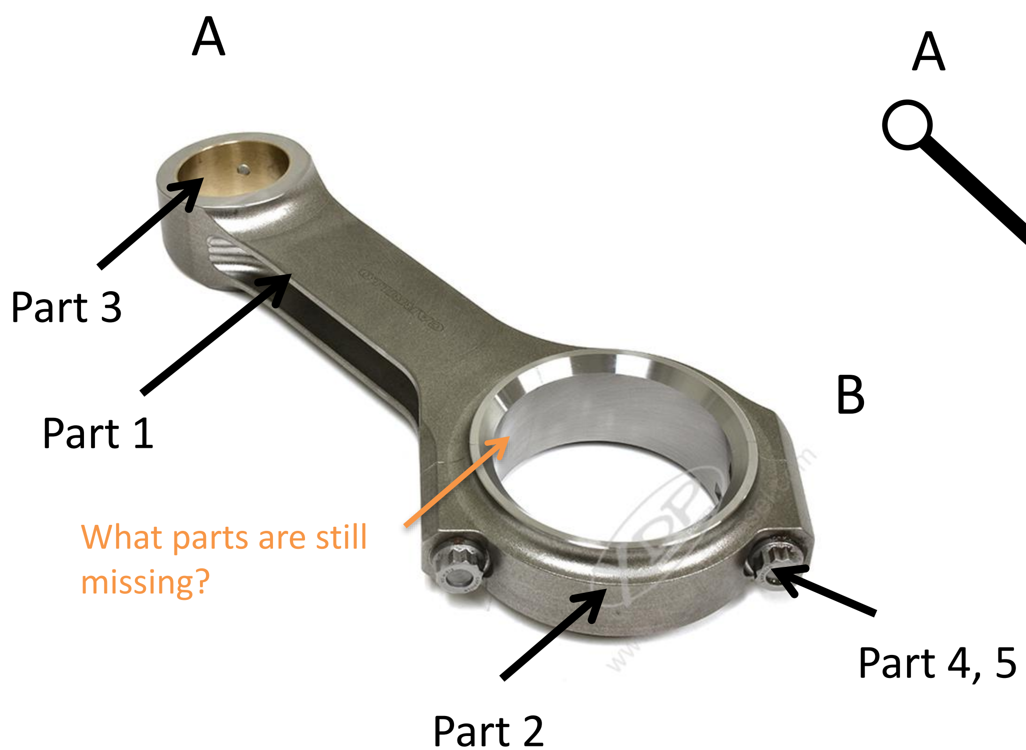
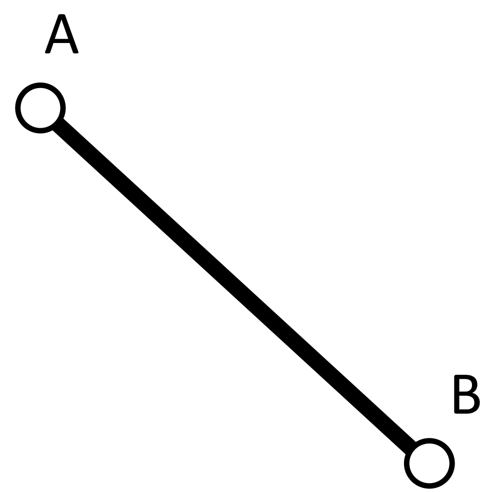


Fig. [http://www.xtremediesel.com/Carrillo-Cummins-Pro-H-Connecting-Rod-with-H-11-Bolts.aspx]

Connecting rod – symbolic representation



Basic terminology and definitions

Types of links (members)

1. Classification by the motion performed relative to the adopted reference system:

- movable (e.g. shaft, piston, rod) - having motion relative to an assumed reference system,
- fixed (e.g. base, frame, body) - having no motion relative to the reference system.

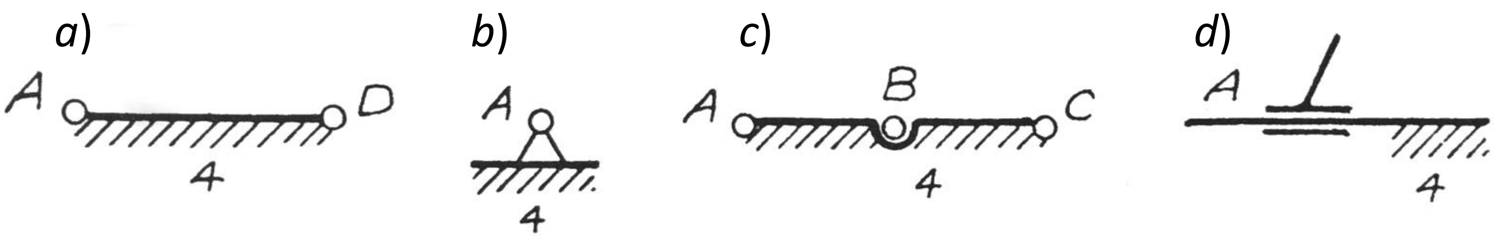


Fig. Fixed link 4 forming the revolute (a, b, c) and sliding (d) pair [Młynarski 1997]

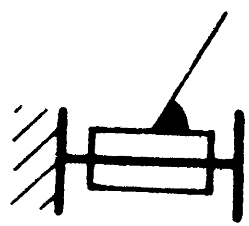


Fig. A fixed link that is part of a spatial revolute pair [Dobrzański 2002]

Basic terminology and definitions

Types of links (members)

2. Classification by the performed role:

- driver (driving) - whose motion is known,
- passive (driven),
- mediating.

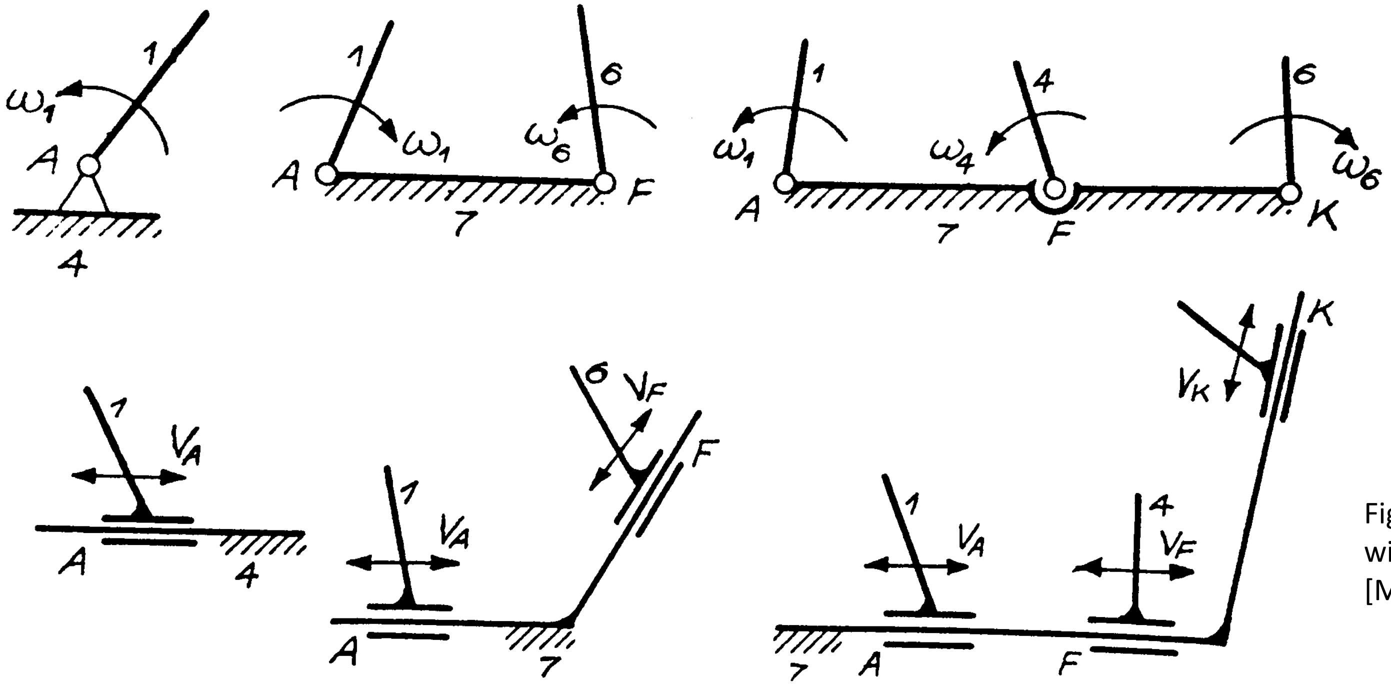


Fig. Driver link marked with an arrow [Młynarski 1997].

Basic terminology and definitions

Types of links (members)

3. Classification by susceptibility:
- rigid - assumed to be non-deformable,
- susceptible, e.g., a spring,
- liquids and gases.

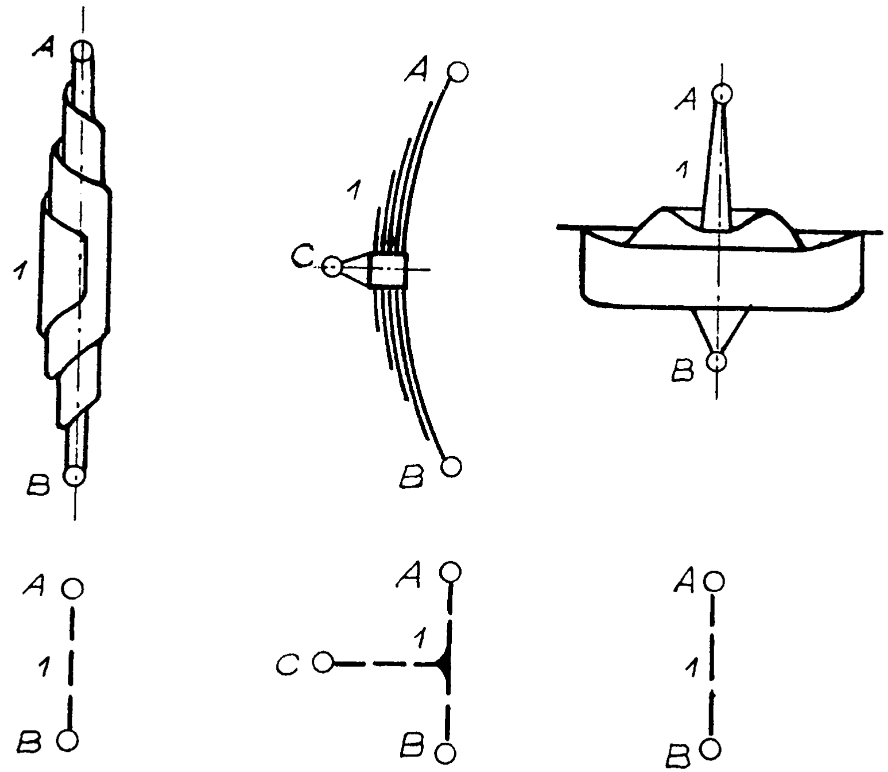


Fig. Susceptible link [Młynarski 1997]

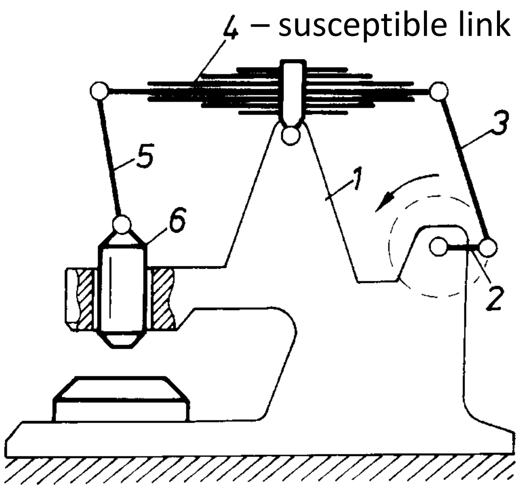


Fig. [Miller 1989]

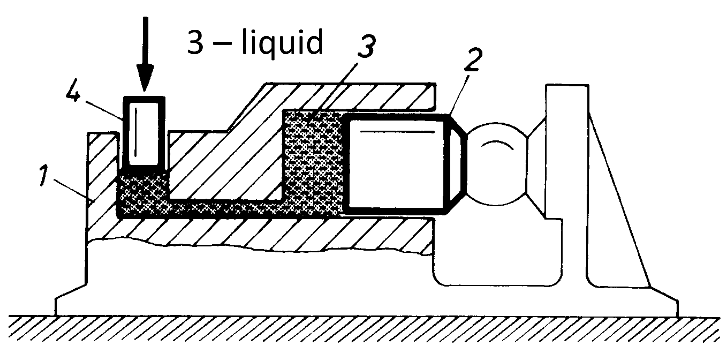
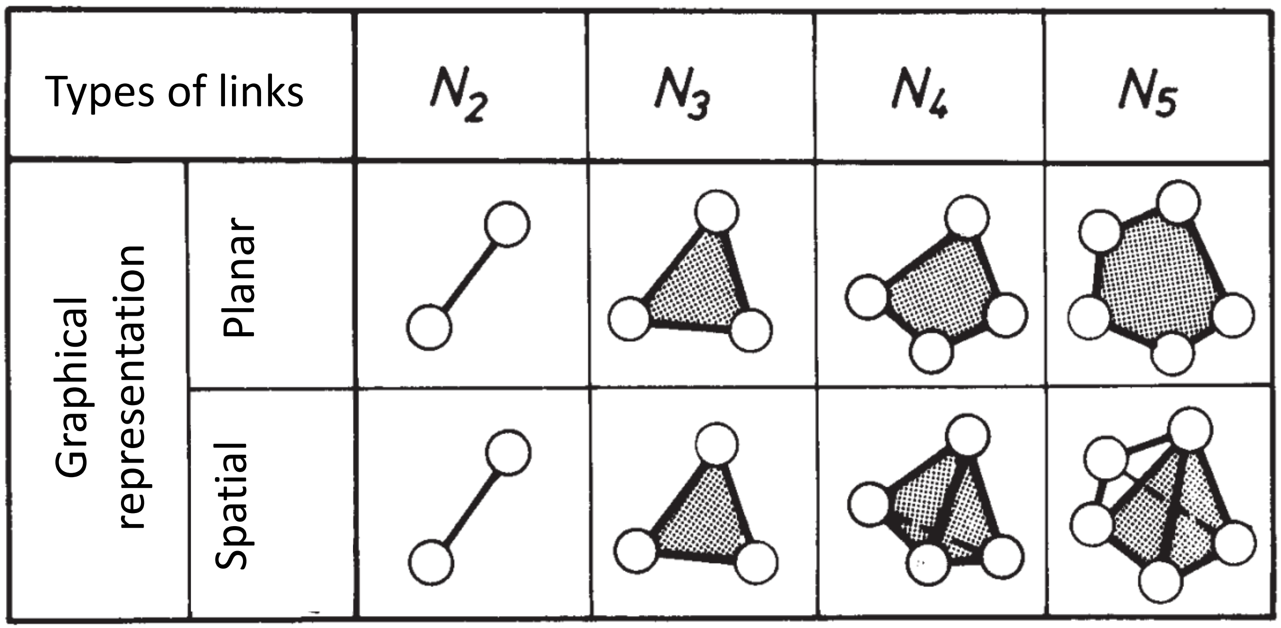


Fig. [Miller 1989]

Basic terminology and definitions

Types of links (members)
According to numer of nodes



Binary link | Ternary link | Quaternary link | Quintuple link

Fig. [Miller 1989]

Basic terminology and definitions

Joint (kinematic pair, pair) - connection between links (two or more), which permits relative motion between them and physically adds some constraint(s) to this relative motion.

Classification of joints according to the :

1. Type of relative motion between the elements.
2. Type of contact between the elements.
3. Type of closure between the elements.
4. Number of links joined.
5. The number of degrees of freedom in relative motion.

Basic terminology and definitions

Classification of joints according to the :

1. Type of relative motion between the elements.

a) ***Revolute*** **(*Turning*)** ***pair****.* When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link. 1 DOF.
b) ***Sliding*** **(*Prismatic, Prism*)** ***pair****.* When the two elements of a pair are connected in such a way that one can only slide relative to the other, the pair is known as a sliding pair. 1 DOF.
c) ***Cylindrical pair.*** Permits angular rotation and an independent translation motion. Thus, the cylindrical pair has two degrees of freedom. 2 DOF.
d) ***Screw*** **(*helical*)** ***pair****.* When the two elements of a pair are connected in such a way that one element can turn about the other by screw threads, the pair is known as screw pair. The lead screw of a lathe with nut and bolt with a nut are examples of a screw pair. 1 DOF.
e) ***Spherical pair.*** When the two elements of a pair are connected in such a way that one element (with spherical shape) turns or swivels about the other fixed element, the pair formed is called a spherical pair. The ball and socket joint, attachment of a car mirror, pen stand etc., are the examples of a spherical pair. 3 DOF.
f) ***Planar pair*** **(*flat*).** It is seldom found in mechanism in its undisguised form, except at a support point. It has three degrees of freedom, that in, two translations and a rotation. 3DOF.

Basic terminology and definitions

Classification of joints according to the :

1. Type of relative motion between the elements.

Theoretically possible:

g) Spherical pair with pivot. The links are connected in such a way that they can perform rotational movements relative to each other about two axes. 2 DOF.

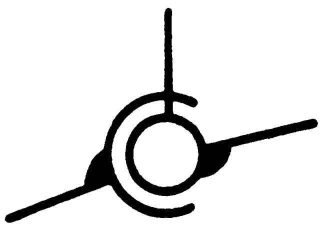


Fig. [Dobrzański 2002]

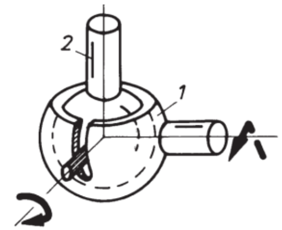


Fig. [Miller 1989]

h) Sphere-cylinder pair. The links are connected in such a way that they can perform rotary motion relative to each other about three axes and rectilinear motion relative to one axis. 4 DOF.

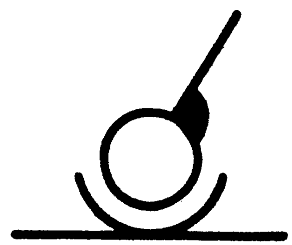


Fig. [Dobrzański 2002]

i) Sphere-plane pair. The legs are connected in such a way that they can perform rotational motion about three axes and translational motion about two axes with respect to each other. 5 DOF.

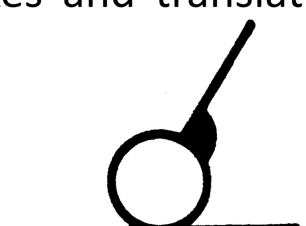


Fig. [Dobrzański 2002]

Basic terminology and definitions

Schematic representation | **Construction** | **Technical realisation**

a) Revolute (Turning) pair – I

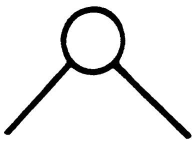


Fig. Plane
[Dobrzański 2002]

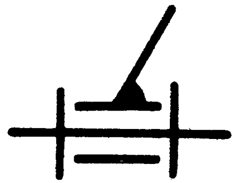


Fig. Spatial
[Dobrzański 2002]

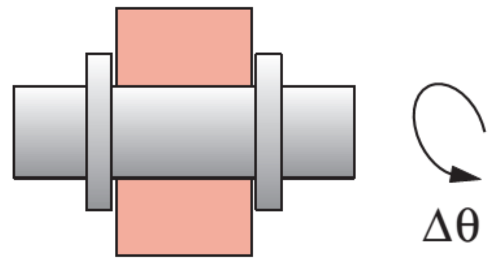


Fig. [Norton 1999]

Fig. [http://airsklep.pl/pl/p/PRZEGUB-OBROTOWY-BEZ-DZWIGNI-30x30-B/181]

b) Sliding (Prismatic, Prism) pair- I

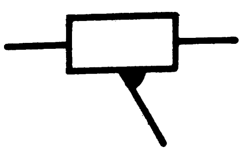


Fig. Plane
[Dobrzański 2002]

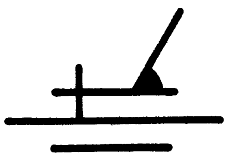


Fig. Spatial
[Dobrzański 2002]

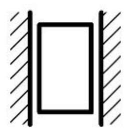


Fig. Plane

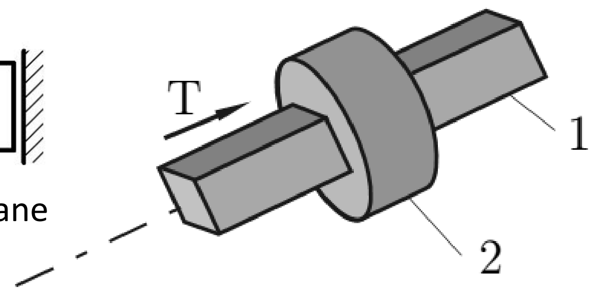


Fig. [Marghitu 2009]

Fig. [http://www.gg-powertransmission.com/splined-shafts-sleeves/standard/]

c) Cylindrical pair - II

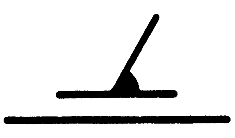


Fig. [Dobrzański 2002]

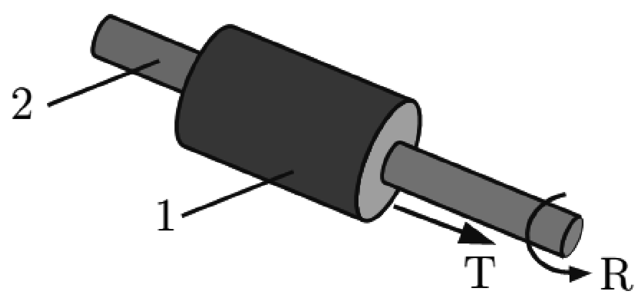


Fig. [Marghitu 2009]

Fig. [http://www.practicalmachinist.com /vb/archive/index.php/t-274723.html]

Basic terminology and definitions

Schematic representation | **Construction** | **Technical realisation**

d) Screw pair – I

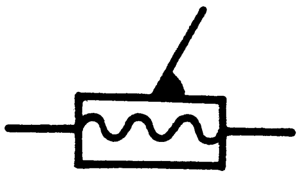


Fig. Wersja 1
[Dobrzański 2002]

Fig. Wersja 2
[Dobrzański 2002]

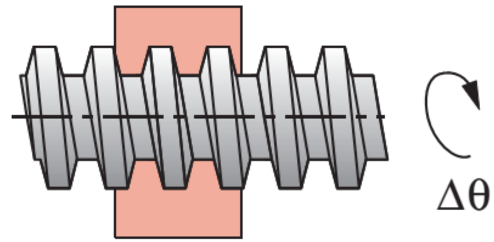


Fig. [Norton 1999]

Fig. [http://www.linearways.in/lead-screws-nuts.html]

e) Spherical pair – III

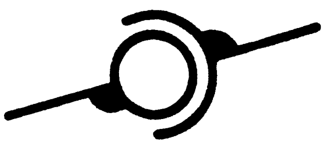


Fig. [Dobrzański 2002]

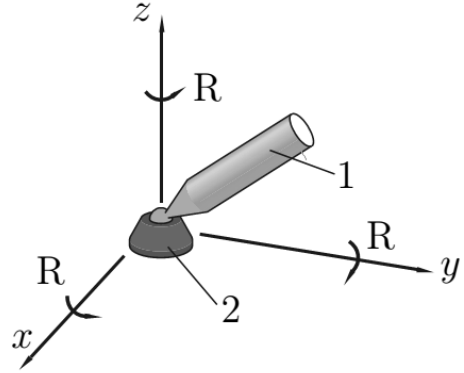


Fig. [Marghitu 2009]

Fig. [https://en.wikipedia.org/wiki/Ball_joint]

f) Flat (planar) pair – III

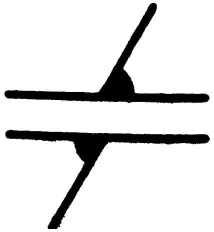


Fig. [Dobrzański 2002]

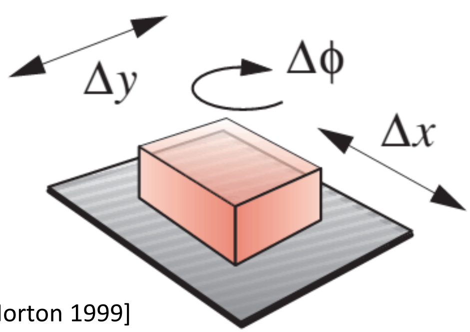


Fig. [Norton 1999]

Fig. [http://sofy24.pl/sofy/162-kanapa-3-osobowa-rozkladana-dawid.html]

Basic terminology and definitions

2. *The kinematic pairs according to the type of contact between the elements may be classified as discussed below* (*Reuleaux*)*:*

a) *Lower pair.* When the two elements of a pair have a surface contact when relative motion takes place and the surface of one element slides over the surface of the other, the pair formed is known as lower pair. Examples of lower pairs (in technical realizations): revolute, sliding, screw, cylindrical, sphere and planar.

b) *Higher pair.* When the two elements of a pair have (theoretically) a line or point contact when relative motion takes place and the motion between the two elements is partly turning and partly sliding, then the pair is known as higher pair. Examples of higher pairs having 2 DOF: gear (half) joint (gear contact) and cam-follower (half) joint (rolling-sliding contact).

Fig. Gear wheels [http://science.howstuffworks.com/transport/engines-equipment/gear2.htm]

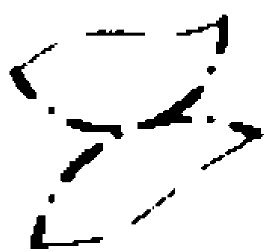


Fig. Number of degrees of freedom, schematic representation and symbolic representation of gear joint [Gronowicz 2003]

Fig. Cam and follower [https://www.cgtrader.com/free-3d-models/industrial/part/cam-follower-mechanism-with-spring]

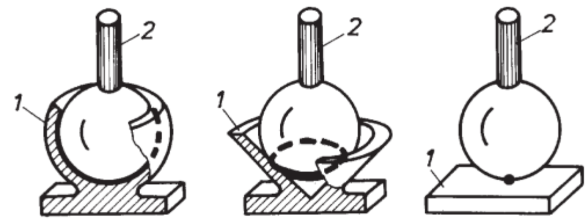


Fig. Theoretical constructions of spherical pair [Miller 1989]

Fig. Number of degrees of freedom and schematic representation of cam and follower [Gronowicz 2003]

Basic terminology and definitions

3. *The kinematic pairs according to the type of closure between the elements may be classified as discussed below :*

a) *Self closed pair.* When the two elements of a pair are connected together mechanically in such a way that only required kind of relative motion occurs, it is then known as self closed pair. The lower pairs are self closed pair.

b) *Force – closed pair.* When the two elements of a pair are not connected mechanically but are kept in contact by the action of external forces, the pair is said to be a force-closed pair. The cam and follower in an example of force closed pair, as it is kept in contact by the force exerted by spring and gravity.

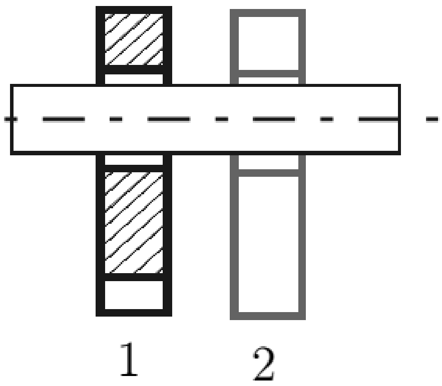
Fig. Cam and follower - forced closed
[https://www.cgtrader.com/free-3d-models/industrial/part/cam-follower-mechanism-with-spring]

Fig. Spherical pair – self closed
[https://en.wikipedia.org/wiki/Ball_joint]

Basic terminology and definitions

4. Classification of joints according to the number of links joined.

The order of a joint is defined as the number of links joined minus one. The combination of two links has order one and it is a single joint.



one-pin joint

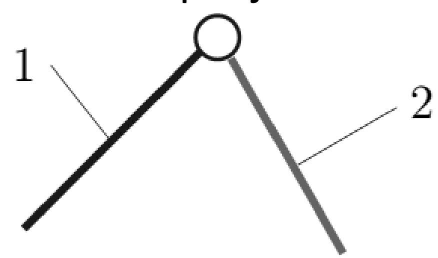
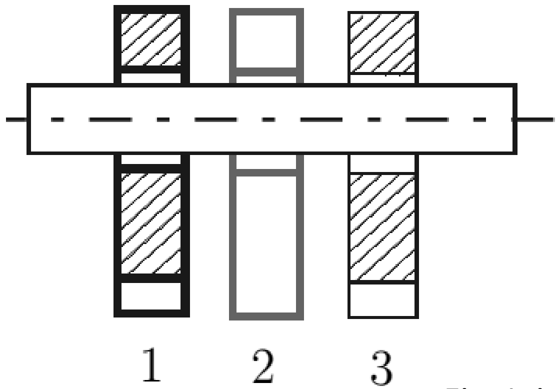


Fig. Joint of order one
[Marghitu 2009]



two-pin joints

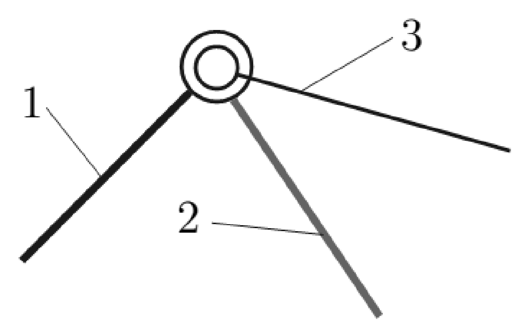


Fig. Joint of order two (multiple joints)
[Marghitu 2009]

Basic terminology and definitions

5. Classification of joints according to the the number of degrees of freedom in relative motion.

This is the most common classification of kinematic pairs. The class of a pair is defined as the number* of left degrees of freedom . A revolute and sliding pair is thus class I, a cylindrical pair class II, a spherical pair class III, etc.

There is also yoke joint (pin in slot) in mechanisms that have 2 DOF not previously presented.

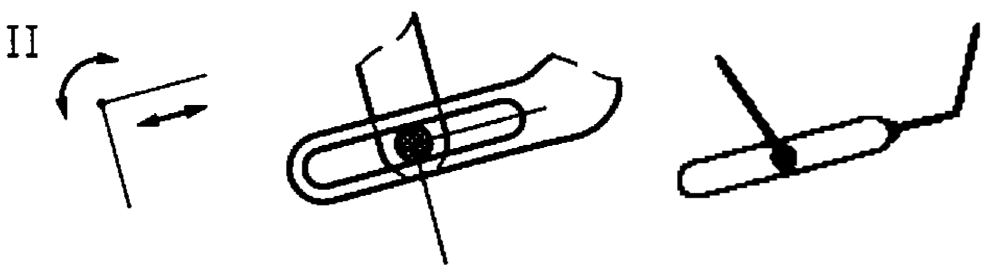


Fig. Number of degrees of freedom, schematic and symbolic representation of the yoke joint [Gronowicz 2003]

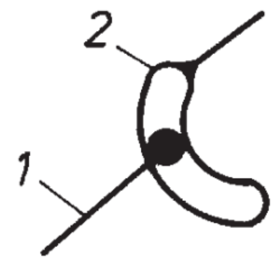


Fig. [Miller 1989]

* Another approach to determining the class of kinematic pairs is also used. The number of removed degrees of freedom determines the class of the pair, so a revolute pair is class V.

Basic terminology and definitions

Number of degrees of freedom (DOF)

Felis, Jaworowski, Cieślik – *The number of degrees of freedom of a rigid body is the number of independent generalized coordinates that determine its position in space with respect to a chosen coordinate system*.

The motion of a floating rigid body in space can be represented as three independent translational motions (relative to X, Y, Z) and three rotational motions (relative to X, Y, Z).

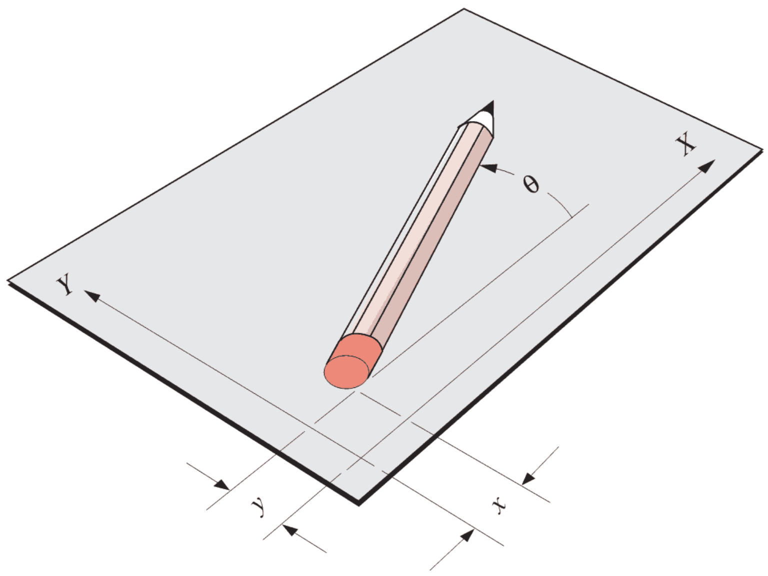


Fig. Rigid body in 2D space (plane) - 3 DOF [Norton 1999]

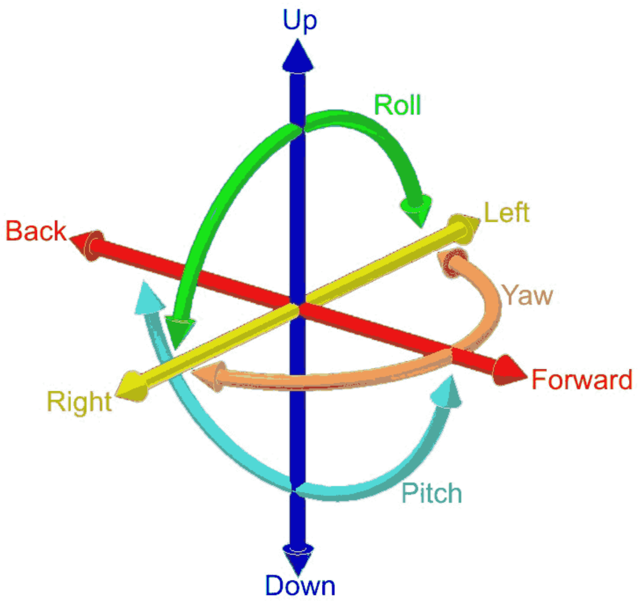


Fig. Rigid body in 3D space - 6 DOF [https://en.wikipedia.org/wiki/Six_degrees_of_freedom]

Basic terminology and definitions

Number of degrees of freedom (DOF)

Other approaches to understand DOF

- The number of inputs that need to be provided in order to create a predictable output;
- Number of independent coordinates needed to define the position of the element/mechanism.

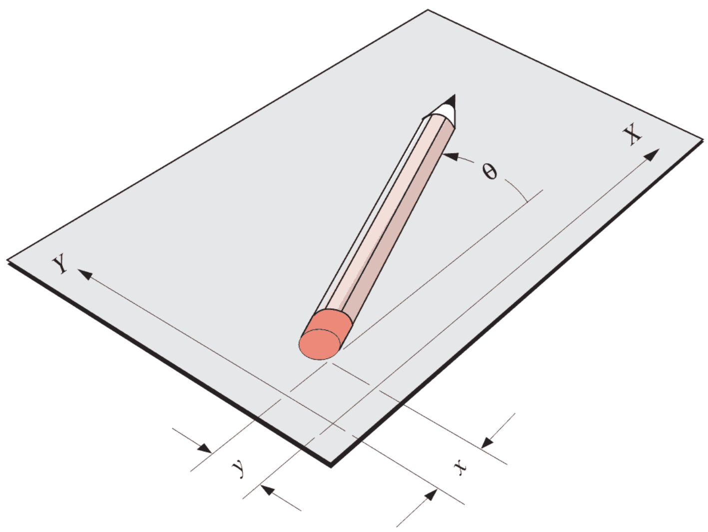


Fig. Rigid body in 2D space (plane) - 3 DOF
[Norton 1999]

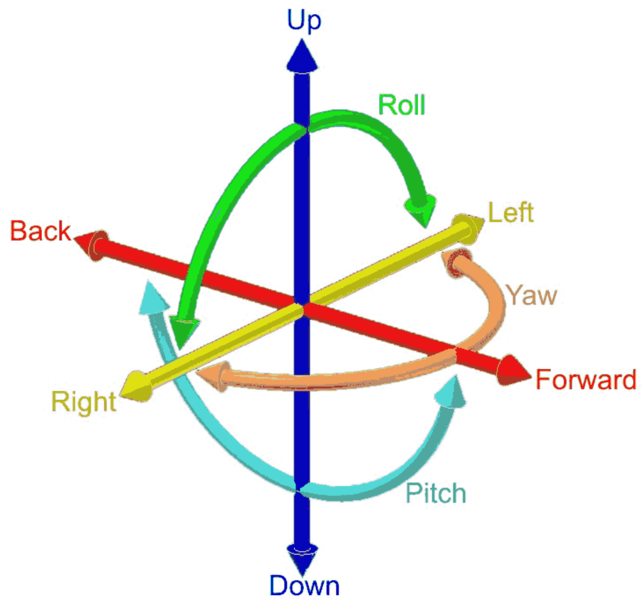


Fig. Rigid body in 3D space - 6 DOF
[https://en.wikipedia.org/wiki/Six_degrees_of_freedom]

Basic terminology and definitions

Mobility *M* (degrees of freedom) of a mechanism

This is the number of degrees of freedom that links have relative to the base (fixed link).
In other words, the mobility of a mechanism can be defined as the number of drives that must be applied for its motion to be determinate.

The formula for calculating the mobility of planar mechanisms, called the Kutzbach criterion, is as follows:

$$M = 3(n - 1) - 2\,p_1 - p_2$$

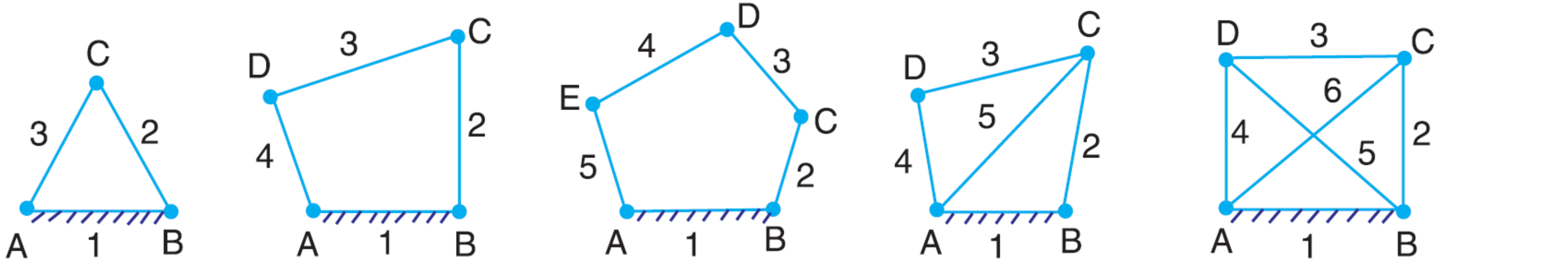
M = mobility,
n = number of links including frame,
p_1 = number of joints of class I (with 1 DOF),
p_2 = number of joints of class II (with 2 DOF).

and for spatial mechanisms :

$$M = 6(n - 1) - 5\,p_1 - 4\,p_2 - 3\,p_3 - 2\,p_4 - p_5$$

Basic terminology and definitions

Motility of plane mechanisms - examples



(a) Three-bar mechanism. (b) Four bar mechanism. (c) Five bar mechanism. (d) Five bar mechanism. (e) Six bar mechanism.

Fig. [Khurmi 2005]

The number of degrees of freedom or mobility (M) for some simple mechanisms having no higher pairs (i.e. $p_4 = 0$) as shown in figure are determined as follows:

a) The mechanism has three links and three binary joints i.e. $n = 3$ and $p_5 = 3$

$$M = 3\ (3 - 1) - 2 \times 3 = 0$$

b) The mechanism has four links and four binary joints i.e. $n = 4$ and $p_5 = 4$

$$M = 3\ (4 - 1) - 2 \times 4 = 1$$

c) The mechanism has five links and five binary joints i.e. $n = 5$ and $p_5 = 5$

$$M = 3\ (5 - 1) - 2 \times 5 = 2$$

d) The mechanism has five links and four binary joints (because there are two joints at *B* and *D*, and two ternary joints at *A* and *C*) i.e. $n = 5$ and $p_5 = 6$

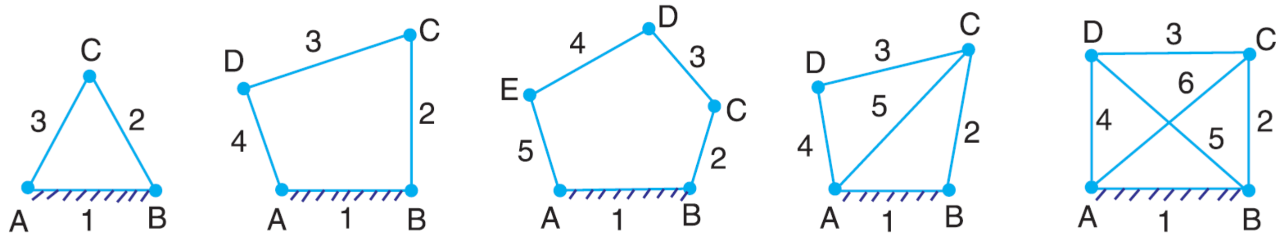
$$M = 3\ (5 - 1) - 2 \times 6 = 0$$

d) The mechanism has six links and eight binary joints (because there are four ternary joints at *A*, *B*, *C* and *D*) i.e. $n = 6$ and $p_5 = 8$

$$M = 3\ (6 - 1) - 2 \times 8 = -1$$

Basic terminology and definitions

Motility of plane mechanisms - examples



Rys. [Khurmi 2005]

(*a*) Three-bar mechanism. (*b*) Four bar mechanism. (*c*) Five bar mechanism. (*d*) Five bar mechanism. (*e*) Six bar mechanism.

It may be noted that:

- When $M = 0$, then the mechanism forms a structure and no relative motion between the links is possible Fig. *a* and *d*;
- When $M = 1$, then the mechanism can be driven by a single input motion, as shown Fig. *b*;
- When $M = 2$, then two separate input motions are necessary to produce unambiguous motion for the mechanism, as shown in Fig. *c*;
- When $M = -1$ or less, then there are redundant constrains in the chain and it forms a statically indeterminate structure, as shown in Fig. *e*.

Basic terminology and definitions

$M \leq 0$ Mechanism with zero, or negative, degrees of freedom are termed locked mechanisms. These mechanisms are unable to move and form a structure.

$M > 1$ Mechanism with multiple degrees of freedom need more than one driver to precisely operate them. In general, multi-degree of freedom mechanism offer greater ability to precisely position a link. There is no requirement that a mechanism have only 1 DOF, although that is often desirable for simplicity of drive control.

Basic terminology and definitions

Kutzbach's–Gruebler's formula for mechanism mobility does not take into account the specific geometry (size or shape) of the links, only the connectivity of links and the type of joints (connections). **Grübler equation not always works.**

Redundand DOF (idle DOF)

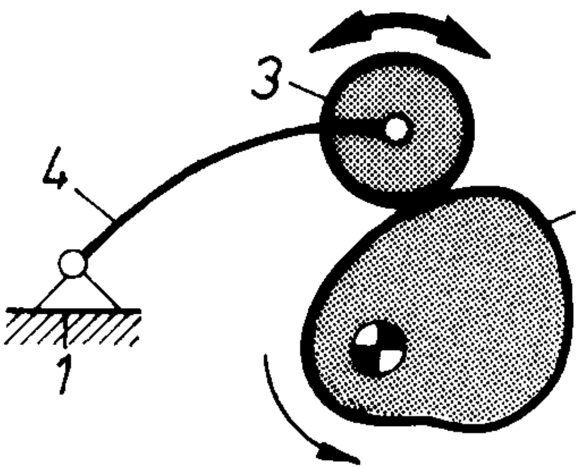
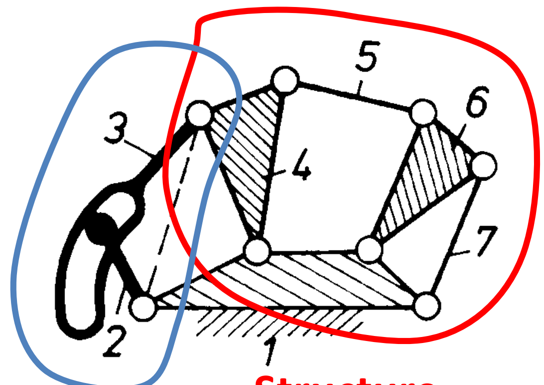


Fig. [Miller 1989]

$M_{\text{real}} = 1$

$M_{\text{equation}} = 3 \times 3 - 2 \times 3 - 1 = 2$

Incomplete mobility



Mechanism

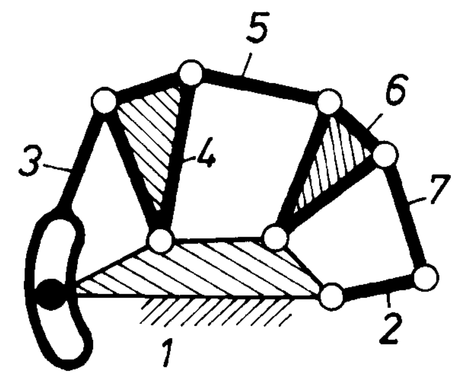
$\mathbf{M_{\text{real}} = 1}$

Structure

$\mathbf{M_{\text{real}} = 0}$

$M_{\text{equation}} = 3 \times 6 - 2 \times 8 - 1 = 1$

Complete mobility

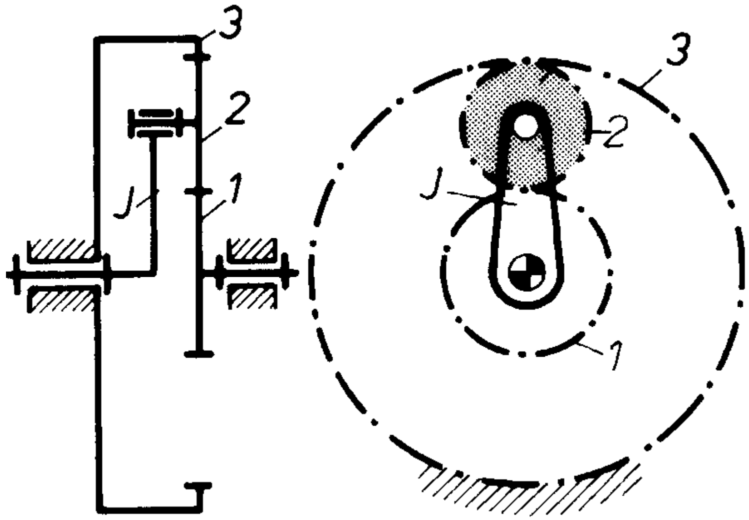


$M_{real} = 1$

$M_{\text{equation}} =$

$3 \times 6 - 2 \times 8 - 1 = 1$

Overconstrained mechanism



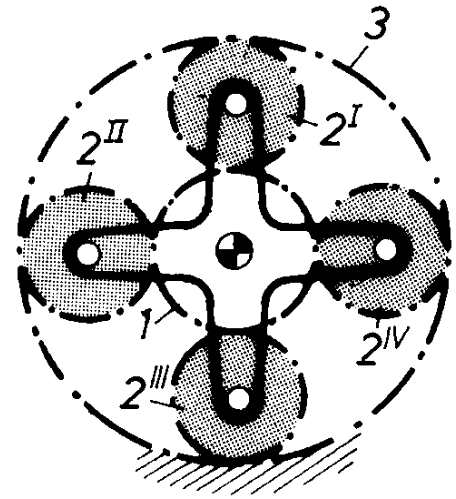


Fig. [Miller 1989]

$M_{real} = 1$
$M_{equation} = 3 \times 3 - 2 \times 3 - 2 = 1$

$M_{real} = 1$
$M_{equation} = 3 \times 6 - 2 \times 6 - 8 = -2$

Basic terminology and definitions

Overconstrained mechanism

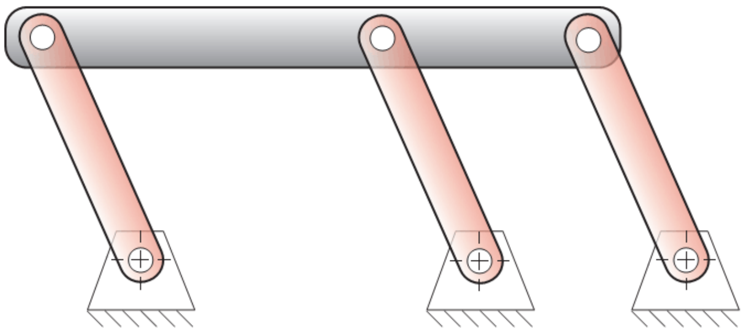


Fig. [Norton 1999]

$M_{real} = 1$

$M_{equation} = 3 \times 4 - 2 \times 6 = 0$

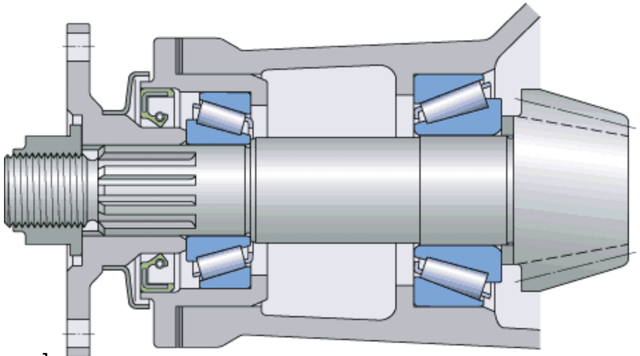


Fig. [www.skf.com]

$W_{real} = 1$

$M_{equation} = 3 \times 1 - 2 \times 2 = -1$

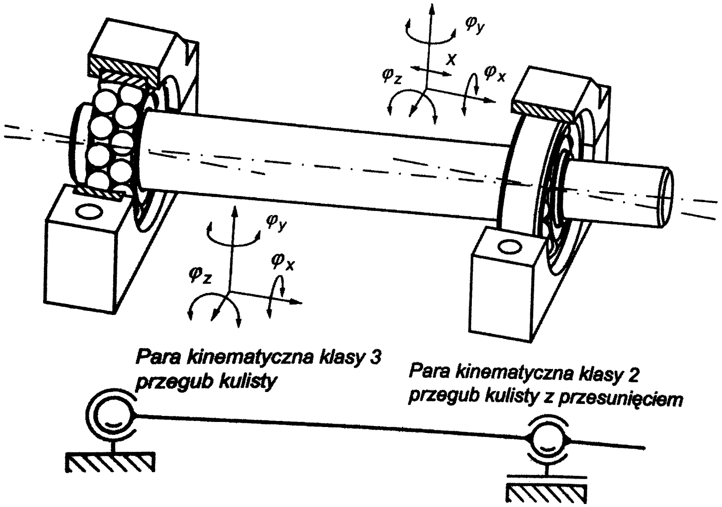


Fig. [Felis 2007]

Classification of mechanisms

There are three types of mechanism classifications:

- structural,
- constructional,
- functional.

Structural classification, that is, division according to structural features, was initiated by the Russian scientist Leonid Vladimirovich Assur (1878-1920) and later developed by other scientists. Ivan Ivanovich Artobolevski (1905-1977) specified the concept of structural group and its class and order [Ceccarelli 2014].

The structural classification adopted in this study divides mechanisms into families, classes, and orders (Fig.). This is not a division that allows unambiguous assignment of all mechanisms to appropriate categories. More accurate methods of classification have been developed, with a greater degree of complexity, the discussion of which is not the scope this presentation.

The advantage of structural classification is the possibility of creating universal methods of structural, kinematic and dynamic analysis.

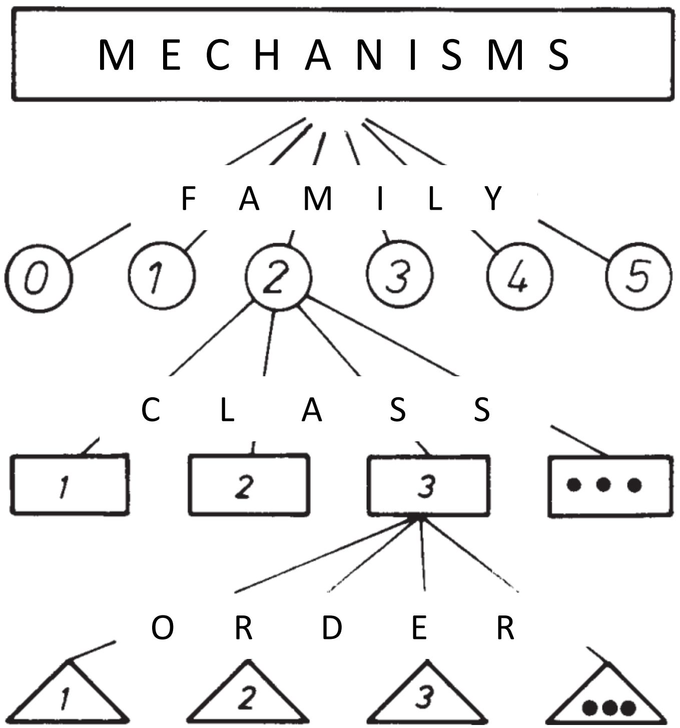


Fig. [Miller 1989]

Family

There are six families from 0 to 5. Membership in a family results from imposed constraints. They are defined relative to the adopted coordinate system jointly for all members. In **family 0**, the links have a total of six degrees of freedom (Figure 1)

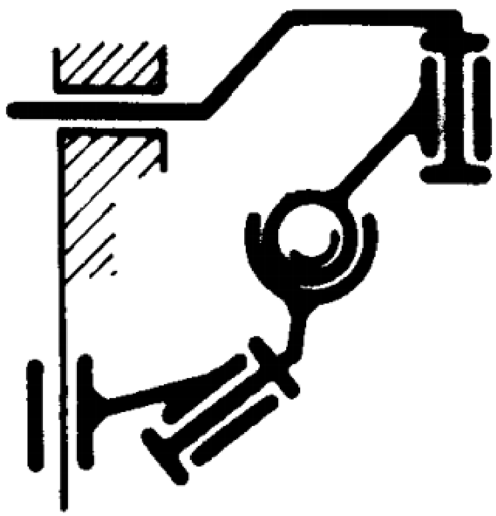


Fig. 1. Family 0 – 6 DOF in total [Miller 1989]

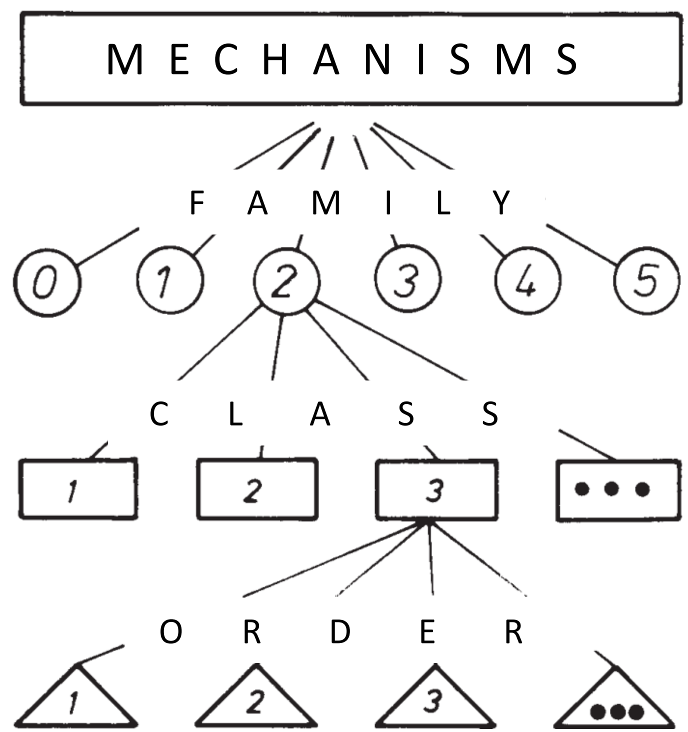
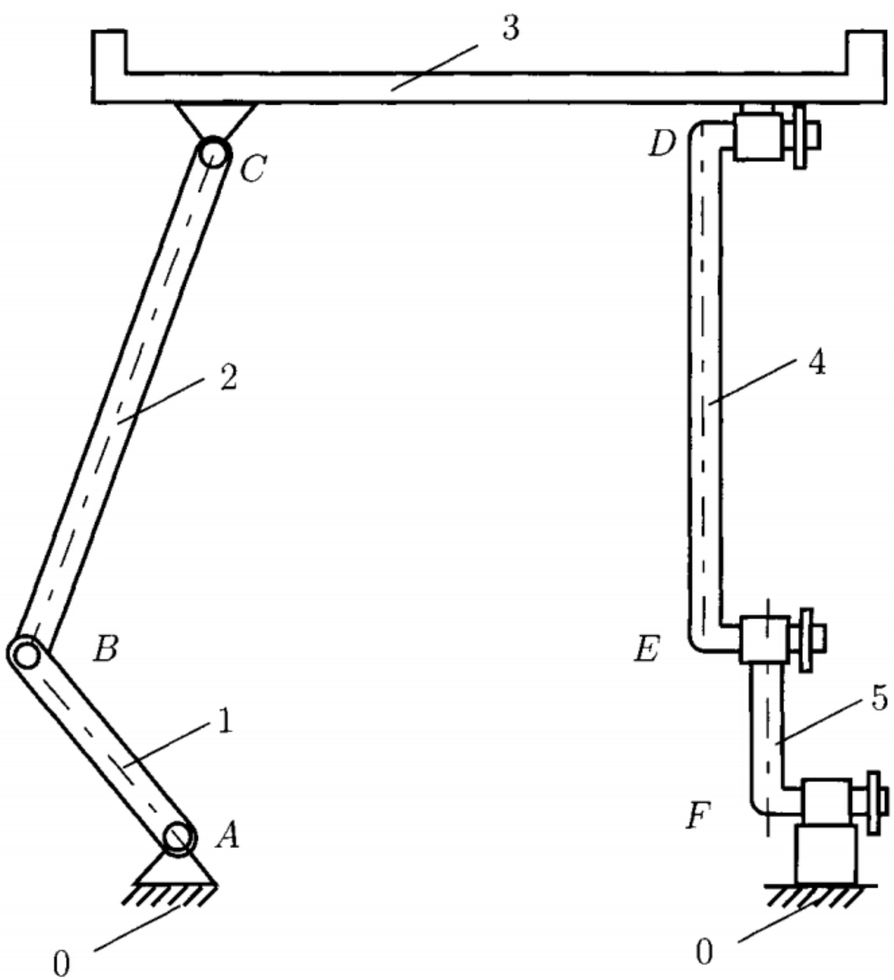


Fig. [Miller 1989]

Classification of mechanisms - structural

Family

Family 1 – all links do not have the same DOF.



Link	T_x	T_y	T_z	R_x	R_y	R_z
0	No	No	No	No	No	No
1	No	No	No	No	No	Yes
2	Yes	Yes	No	No	No	Yes
3	No	Yes	No	No	No	No
4	No	Yes	Yes	Yes	No	No
5	No	No	No	Yes	No	No
					No	

For all links $R_y = \text{No} \Longrightarrow f = 1$.

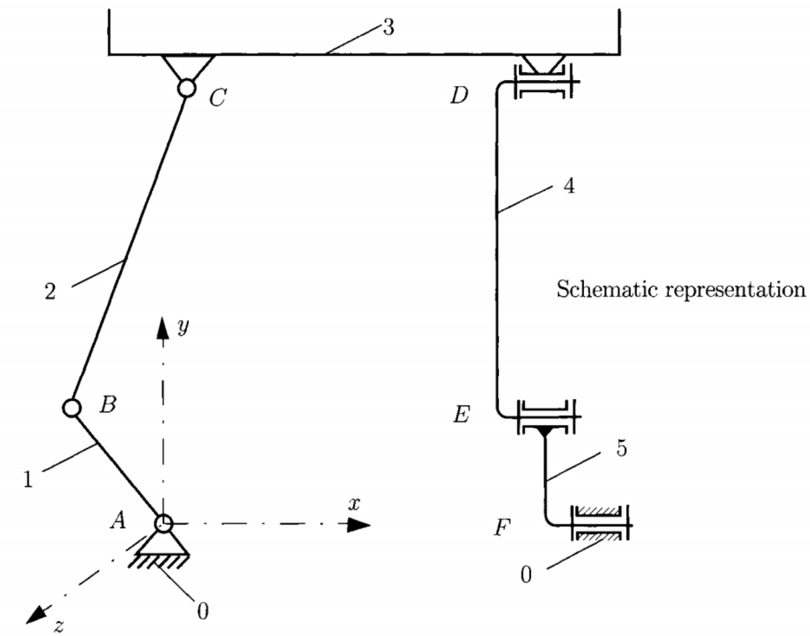


Fig. Family 1 – 5 DOF in total [Marghitu 2005]

Classification of mechanisms - structural

Family

Family 2 – all links have taken away the same DOF. For the mechanism in Figure 2, this is rotation about the vertical axis and translation about the axis perpendicular to the plane of the figure.

Family 3 – all links have taken away the same 3 DOF. For the mechanism in Fig. 3, these are rotation about the vertical and horizontal axes and translation about an axis perpendicular to the plane of the figure. This family includes mainly plane mechanisms (also spherical spatial mechanisms).

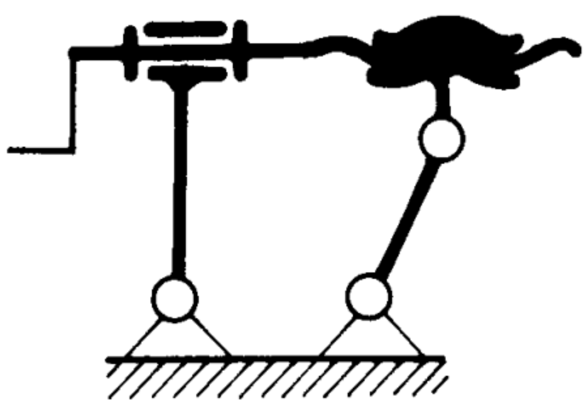


Fig. 2. Family 2 – 4 DOF in total [Miller 1989]

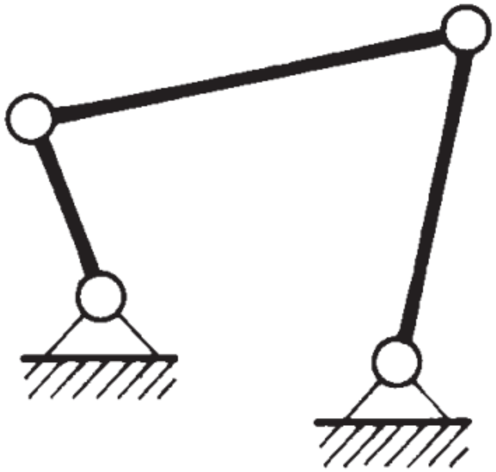


Fig. 3. Family 3 – 3 DOF in total [Miller 1989]

Classification of mechanisms - structural

Family

Family 4 – all links have left the same 2 DOF. For the wedge mechanism in Fig. 1, this is displacement about the horizontal and vertical axes. This family includes sliding pair mechanisms (wedge mechanisms) and screw mechanisms.

Family 5 – link has 1 DOF (Fig. 2). Driver (driving) links belong to this family.

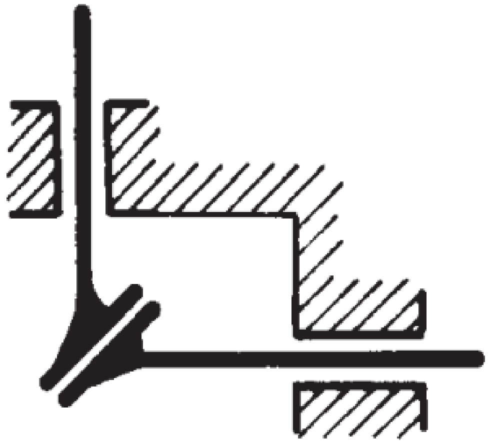


Fig. 1. Family 4 – 2 DOF in total [Miller 1989]

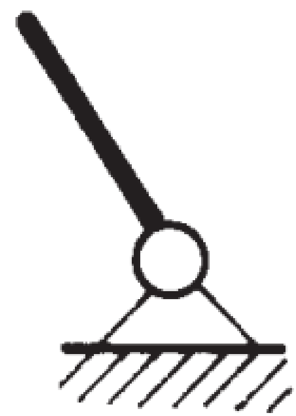


Fig. 2. Family 5 – 1 DOF in total [Miller 1989]

Classification of mechanisms - structural

Class of mechanism (the description presented here refers to plane mechanisms)

The class of mechanism is determined by the class of structural groups (Assur groups) that make up the mechanism. Perform the following steps to determine the class of mechanism:

1. Make a kinematic diagram of the mechanism whose mobility corresponds to the real mobility (remove redundand DOF and links that overconstrained mechanism).
2. If class II joints are present, make an alternative kinematic diagram in which they are replaced by class I joints.
3. Determine the class of the Assur group. The class of the mechanism is equal to the highest class of group present in the mechanism. For some mechanisms, such as geared or complex mechanisms, it is convenient to create a structural diagram before.

Classification of mechanisms - structural

Class of mechanism

A structural (Assur) group is a kinematic chain jointly fulfilling the conditions:

1. When connected to a base (fixed link) of free links, it forms a structure.
2. It is not possible to split it into simpler chains satisfying the first condition.
3. By separating a structural group from an existing mechanism, its mobility cannot be changed.

It follows from (1) that the Assur group has mobility equal to 0 and is obeyed by a structural equation of the form:

$$3n - 2p_1 = 0$$
$$3n = 2p_1$$

From the equation, it is obtained how many links and class I joints the Assur group can have (Table).

Number of links n	2	4	6	...
Number of joints p_1	3	6	9	...

Classification of mechanisms - structural

Class of mechanism

The class of Assur group for 2 links and 3 joints is referred to as second (one internal kinematic pair).

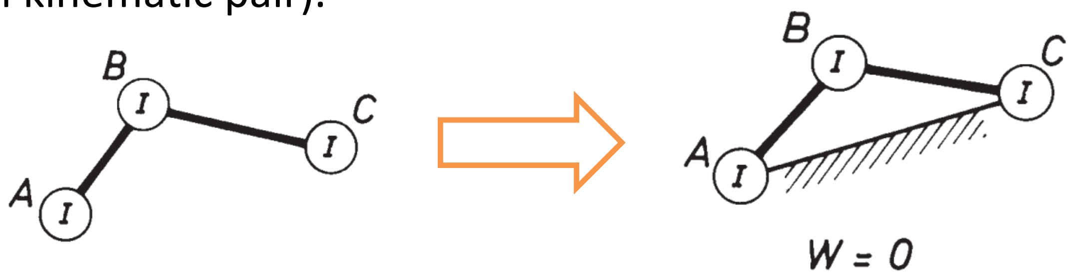


Fig. Example Assur group connected to the base [Miller 1989]

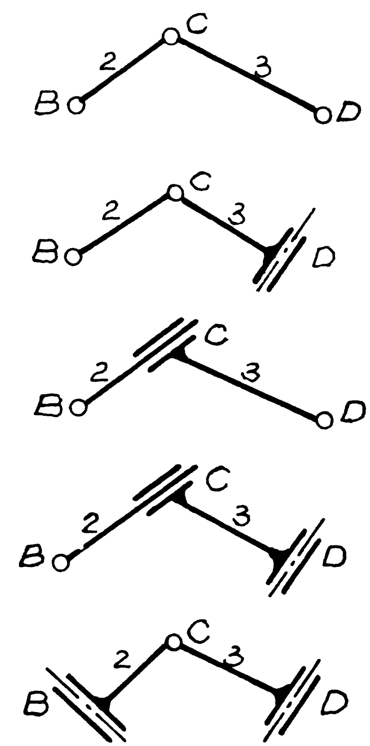


Fig. All variants of a class II group with class I joints [Młynarski 1997]

For other groups consisting of more links, the class of the group is determined by the number of sides of the largest closed polygon of the group*.

*According to some sources [e.g. Siemieniako 1999, Miller 1989], the class of a group is determined by the number of members and kinematic pairs. Below is a table with the class membership of groups according to this rule.

Class of group	*II*	*III*	*IV*	
Number of links *n*	2	4	6	...
Number of joints p_1	3	6	9	...

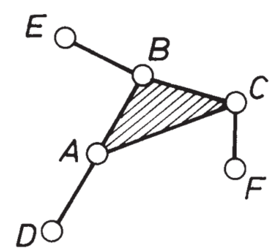
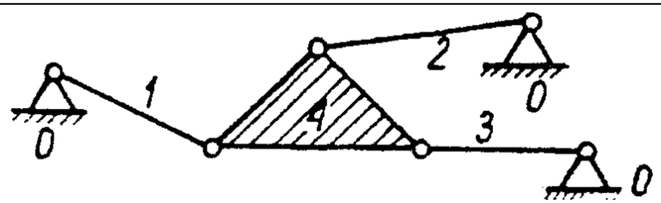
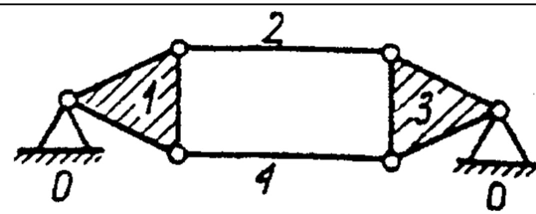
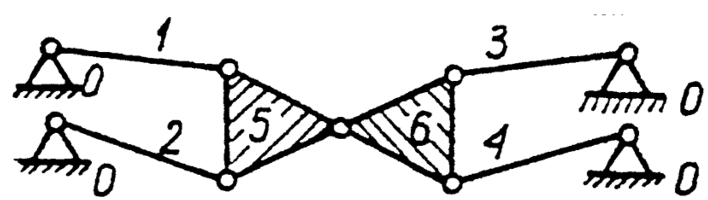
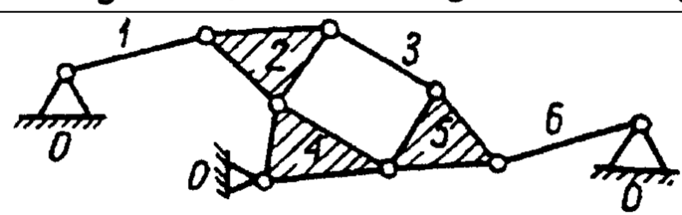


Fig. An example of a class group III [Miller 1989]

Class of mechanism

The two rules presented for determining the class of a group, closed polygon and based on the number of links and joints, are not equivalent (the figures in the table are from [Siemieniako 1999]).

Class of group by		Number of members	Number of joints	Group structural diagram
A closed polygon	Numbers of links and joints			
III	*III*	4	6	
IV	*III*	4	6	
III	*IV*	6	9	
IV	*IV*	6	9	

Class of mechanism

In this classes we will use the closed polygon rule introduced by I. I. Artobolevski [Ceccarelli 2014].Some authors refer to the drive link as a group of class I.

Mechanism order

The order of a mechanism is defined as the number of exterior nodes of a group that, when connected to a base, form a structure.

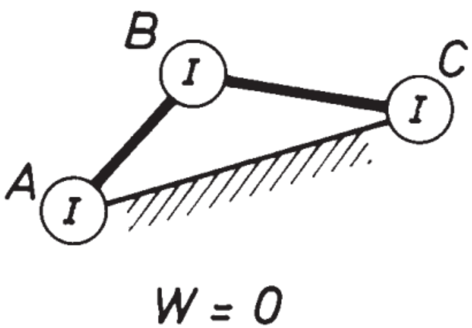


Fig. Group of order 2
[Miller 1989]

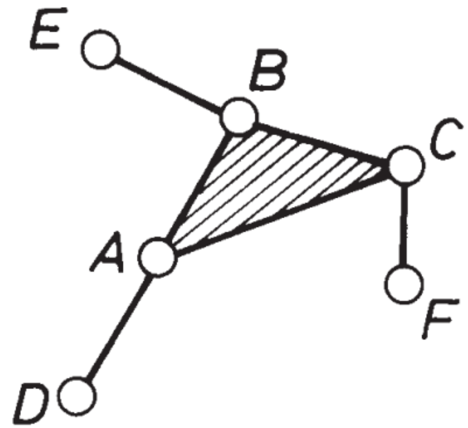
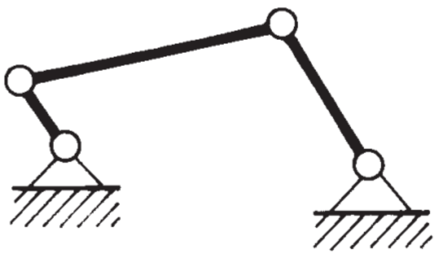


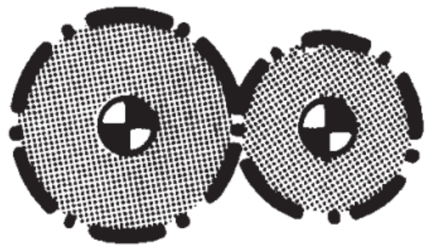
Fig. Group of order 3
[Miller 1989]

Classification of mechanisms - constructional

Constructional classification divides mechanisms based on constructional properties.

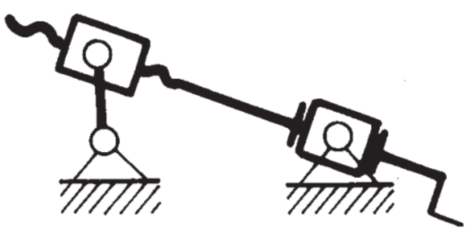


Leverage

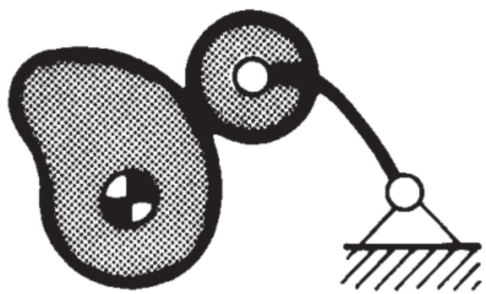


Gear

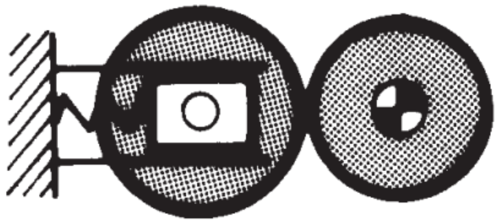
Flexible connector



Screw



Cam



Frictional

Fig. [Miller 1989]

Classification of mechanisms - functional

Functional classification divides the mechanisms due to the tasks performed in machines. Mechanisms according to Artobolevsky are divided into [Mlynarski 1997]: switching on and off, grippers, scales, brakes, stopping, for performing mathematical operations, steering, lever, valves, parallelograms, non-parallelograms, pistons, rotor and eccentric pumps, reels machines, regulators, with swinging discs, presses, power supply, casual power supply, heavy lifting, hammers, locking devices, timing, ratchet and other devices.

Structural analysis of mechanisms

The structural analysis of the mechanism aims to determine its mobility, structural classification and is the starting point for further kinematic and dynamic analysis.

During the analysis, diagrams can be made:

- *kinematic* - depicts the mechanism in a simplified manner, preserving the construction and characteristic geometric sizes of the links necessary for kinematic analysis,
- *structural* - depicts in a symbolic way the connections of the links.

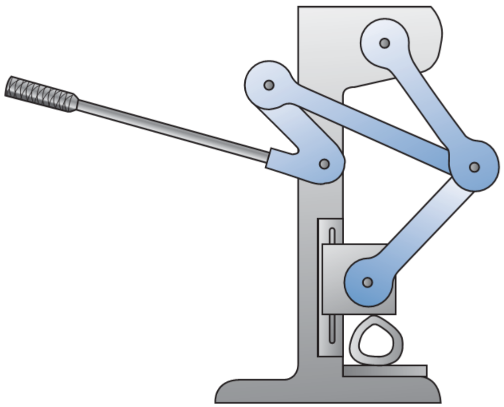


Fig. Constructional diagram of press [Myszka 2012]

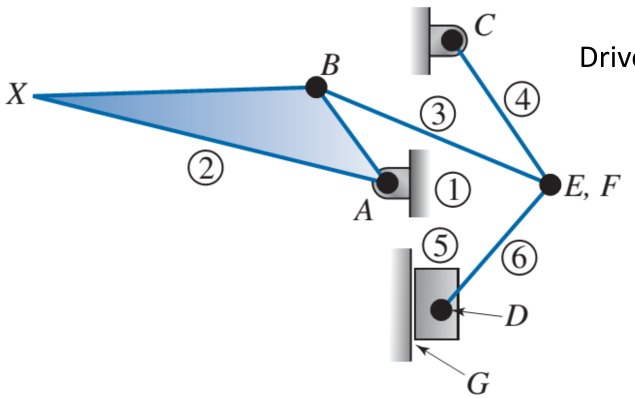


Fig. Kinematic diagram of press [Myszka 2012]

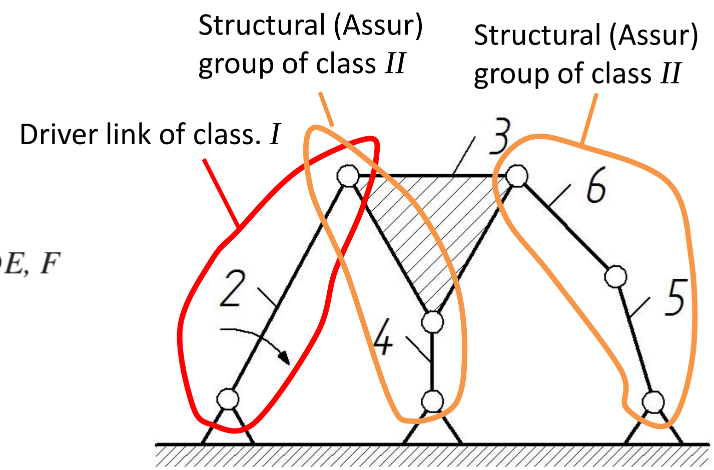


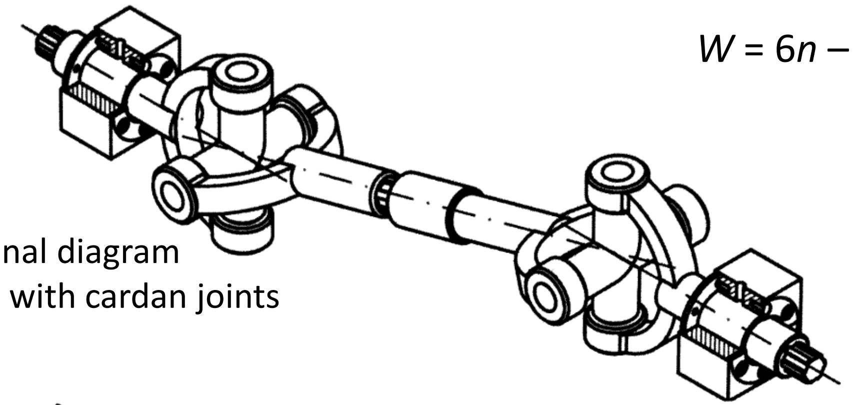
Fig. Structural diagram of press

$M = 3n - 2P_1 - P_2 = 3 \cdot 5 - 2 \cdot 7 = 15 - 14 = 1$

The press mechanism has a mobility of 1 and is classified as family 3, class II, order 2.

What needs to be changed to make it a III class mechanism?

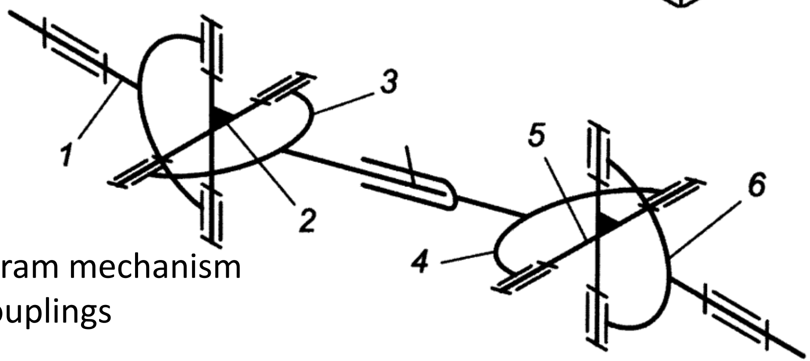
Structural analysis of mechanisms



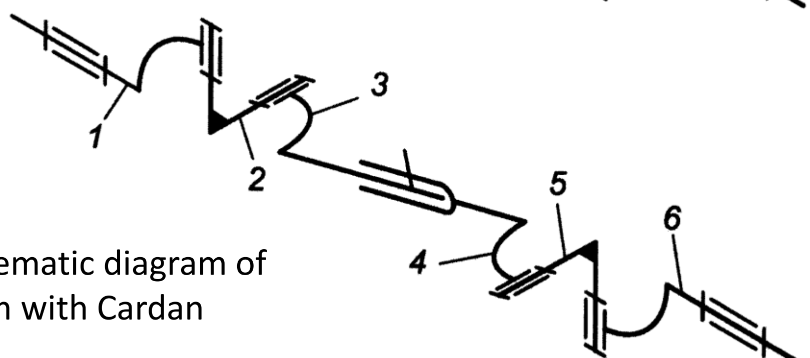
Constructional diagram
mechanism with cardan joints

$$W = 6n - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5 = 6 \cdot 6 - 5 \cdot 7 = 36 - 35 = 1$$

What is the class of the mechanism?



Kinematic diagram mechanism
with Cardan couplings



Equivalent kinematic diagram of
the mechanism with Cardan
couplings

Fig. [Felis, Jaworski 2007]

Linkage mechanisms

Linkage mechanism* is build only from lower pairs except screw pair.

The simplest mechanism of this group is a rotor shaft e.g. of a fan, electric motor supported in bearings,

the basic mechanism, however, is the four bar linkage. A considerable part of linkage mechanisms can be obtained from it by inversion[1] and transformation [2].

Types of links that occur in a four bar linkage:
Crank (driver link) - can make a full rotation and is connected to the base.
Rocker (follower, lever) - performs an oscillating-rotating (pendulum) motion and is connected to the base.
Coupler (connecting arm) - performs a plane motion and is not connected to the base.
Base (frame, fixed link) - performs no motion relative to the assumed reference system.

[1]*Inversion* (*kinematic inversion*) – is a change of base from one link to another e.g. crank with base
[2]*Transformation* – is a change of type of joint e.g. revloute to sliding

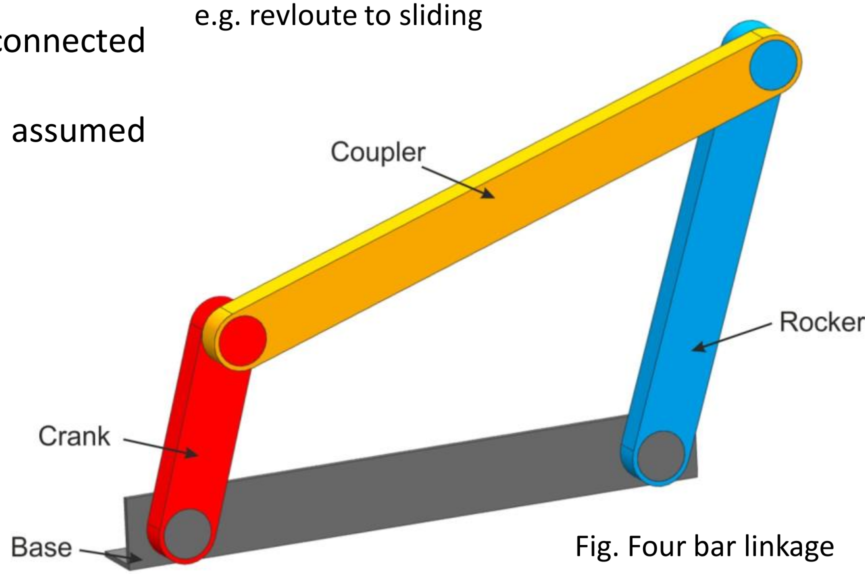


Fig. Four bar linkage

*There is no one definition of linkage mechanism. Presented definition is not common and narrows scope of types of mechanism that can be assigned to this group.
Linkage – *mechanism having only lower pairs* [Uicker 2011]
Linkage – *mechanisms consisting of lower pairs only* or *loosely synonymous to the term mechanism* [Ambekar 2007]
Linkage – *is a mechanism where rigid parts are connected together to form a chain* [Myszka 2012]

Linkage mechanisms

From a practical point of view, it is important to know what conditions must be met by the links of the four bar linkage so that one of them can make a full rotation. In this case, a motor can drive such a link. The answer to this question is given by Grashof's law, which states that for a four-bar linkage, if the sum of the shortest and longest links is not greater than the sum of the remaining two links, at least one of the links will be revolving. Grashof's law (condition) is expressed in the form:

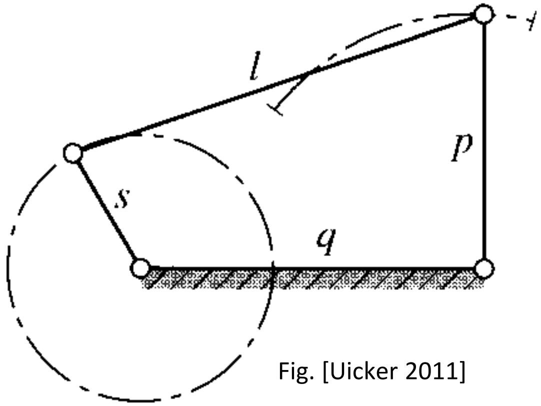


Fig. [Uicker 2011]

$$s + l \leq p + q$$

s = length of shortest link
l = length of longest link
p = length of one remaining link
q = length of other remaining link

Reuleaux approach the problem somewhat differently but, of course, obtains the same results. The conditions are:

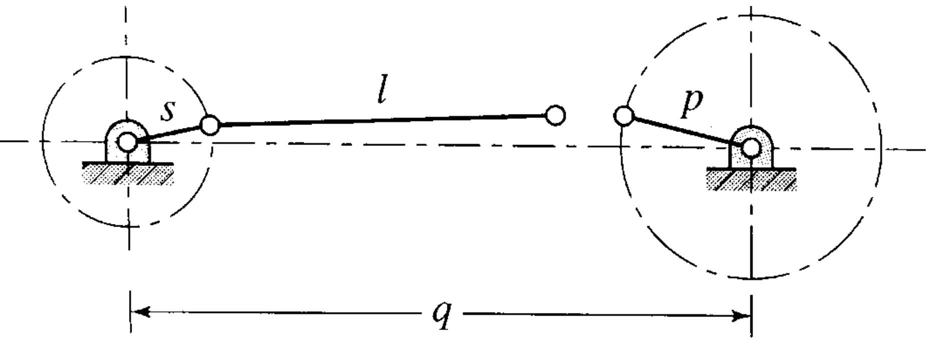
$s + l + p \geq q$
$s + l - p \leq q$
$s + q + p \geq l$
$s + q - p \leq l$

The result is obviously the same.

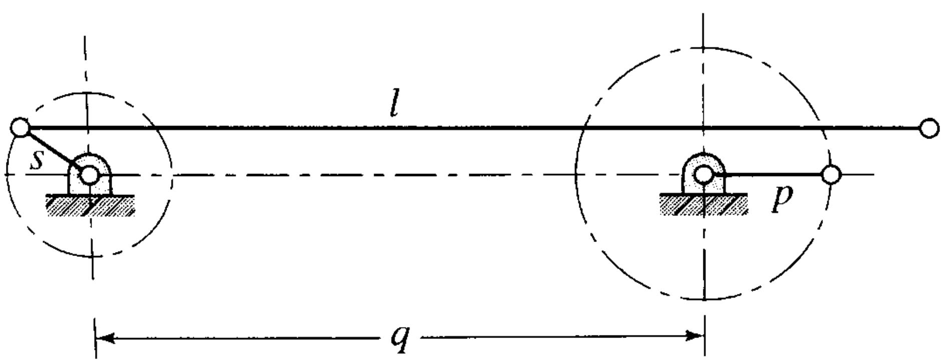
Linkage mechanisms

These four condition are illustrated below by demonstrating what happens if the conditions are not met.

Implementation is not possible

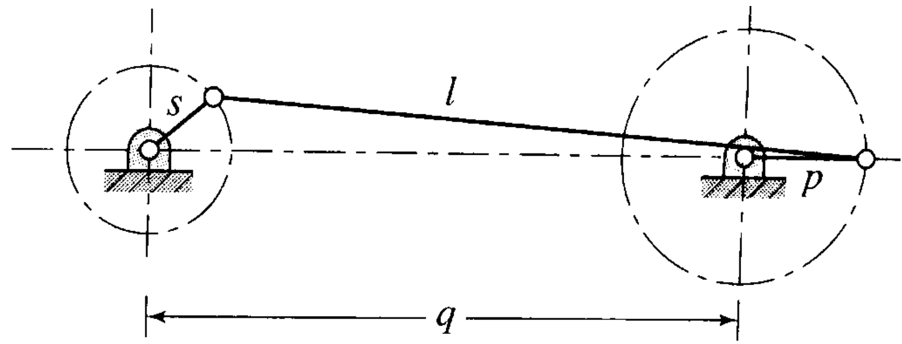


$$s + l + p < q$$

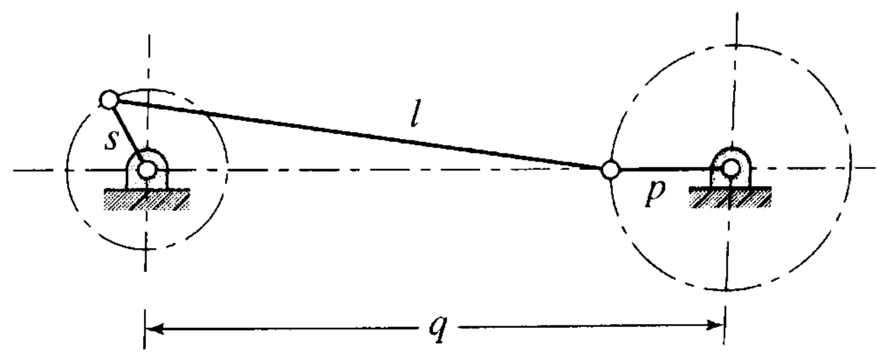


$$s + q + p < l$$

Link *s* is not a crank



$$s + l - p > q$$

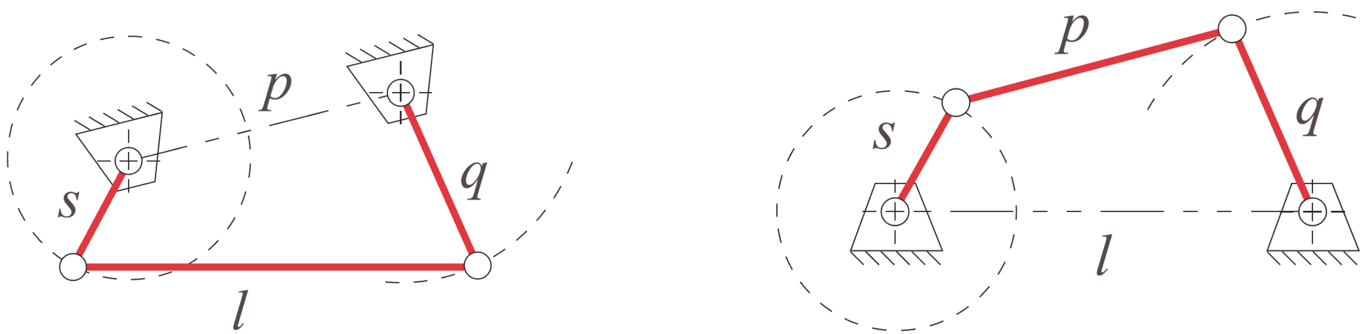


$$s + q - p < l$$

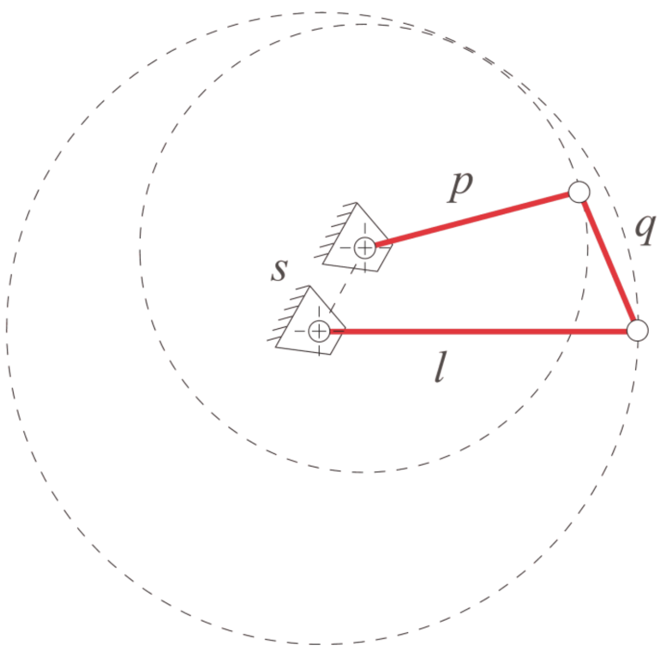
Fig. [Uicker 2011]

Linkage mechanisms

Depending on the functions performed by the links (making inverstion), three types of mechanisms are obtained.



Crank-rocker



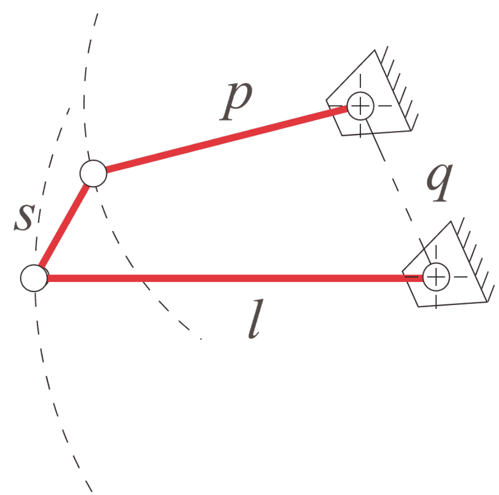


Fig. [Norton 1999]

Double crank

Double rocker

Linkage mechanisms

By making a transformation of the revloute joint to the sliding joint of the four bar mechanism, a slider-crank mechanism is obtained (Fig. 1). It is commonly used in internal combustion engines and compressors.

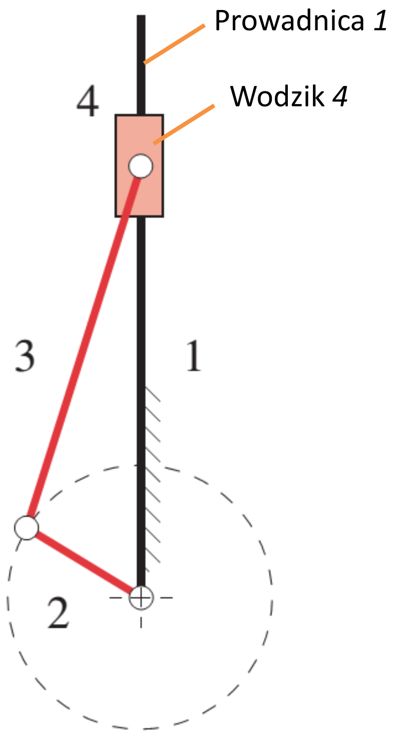


Fig. 1. Slider-crank linkage
[Norton 1999]

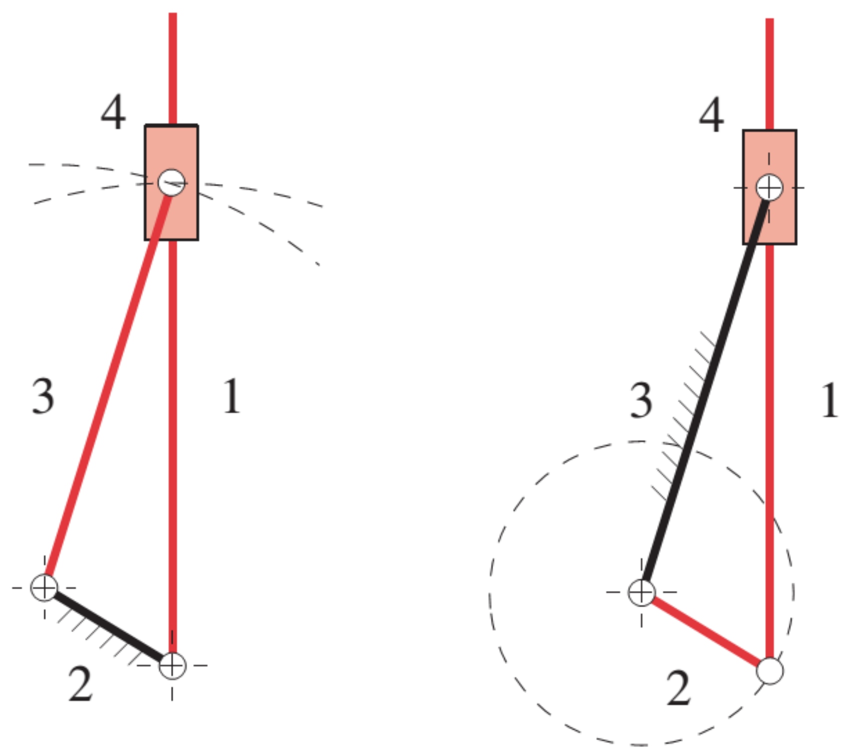


Fig. 2. Inversion of the slider-crank linkage
[Norton 1999]

Linkage mechanisms

Converting kinematic pairs of class 2 to pairs of class 1 makes it possible to carry out a kinematic analysis using the methods used for linkage mechanisms. The created equivalent mechanism must be kinematically equivalent to the real one at the given position. An additional member is introduced so that the mobility of the mechanism remains unchanged. In the case of cam mechanisms, the ends of the link are at the center of the adjoining curves.

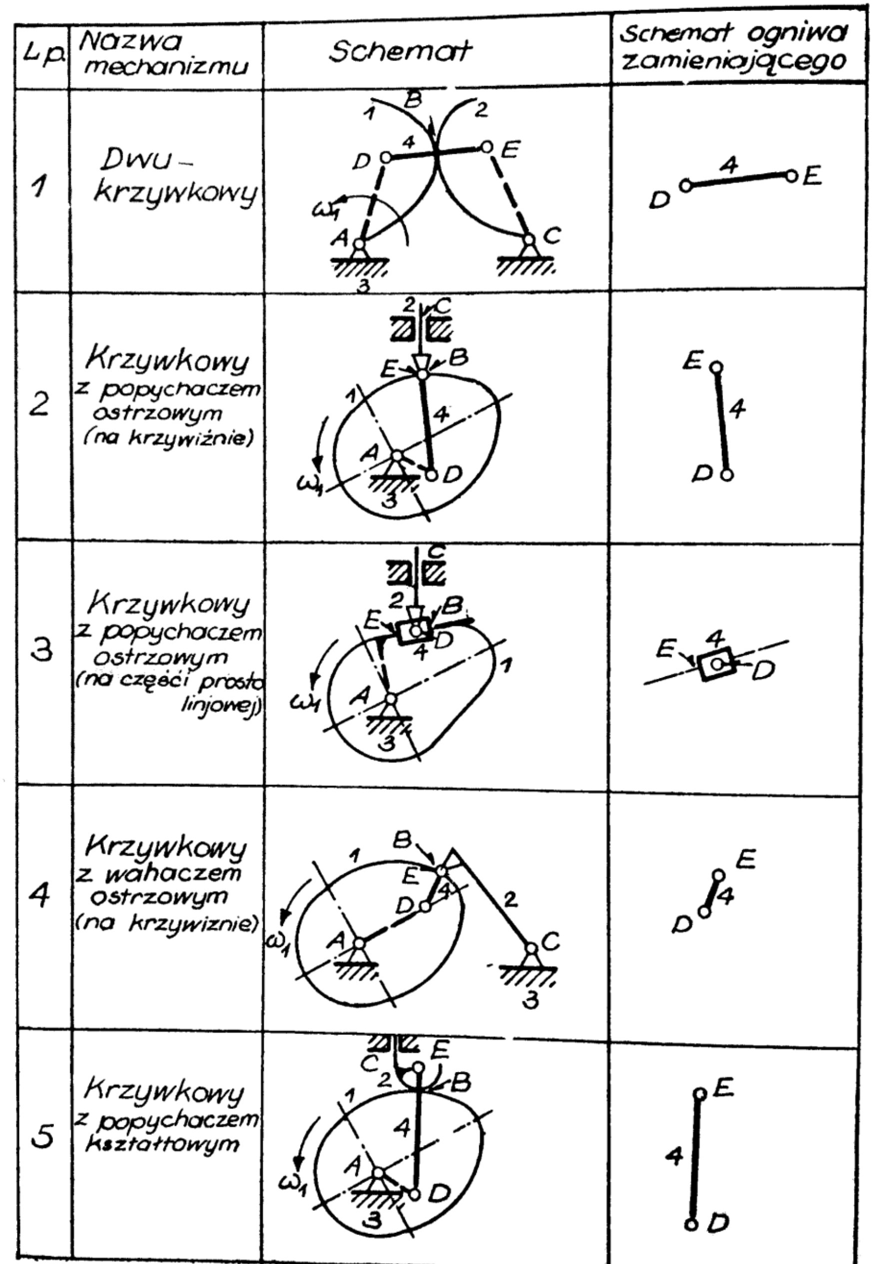


Fig. [Młynarski 1997]

Linkage mechanisms

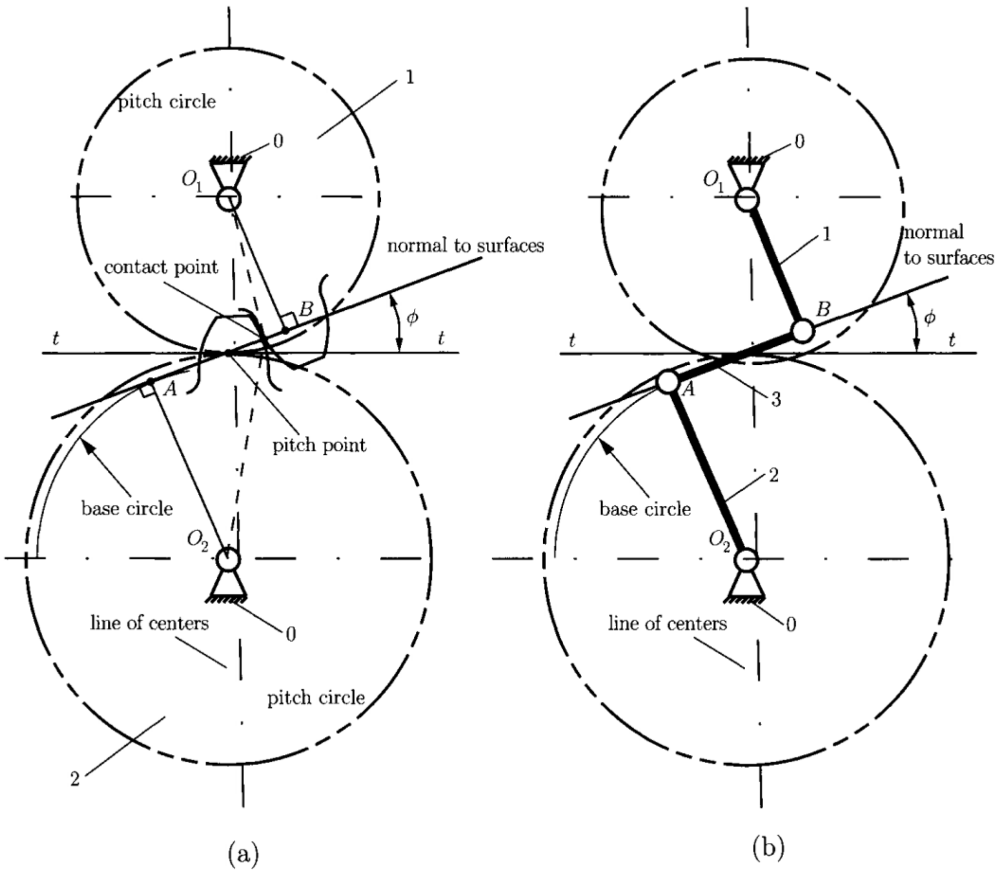


Fig. [Marghitu 2005]

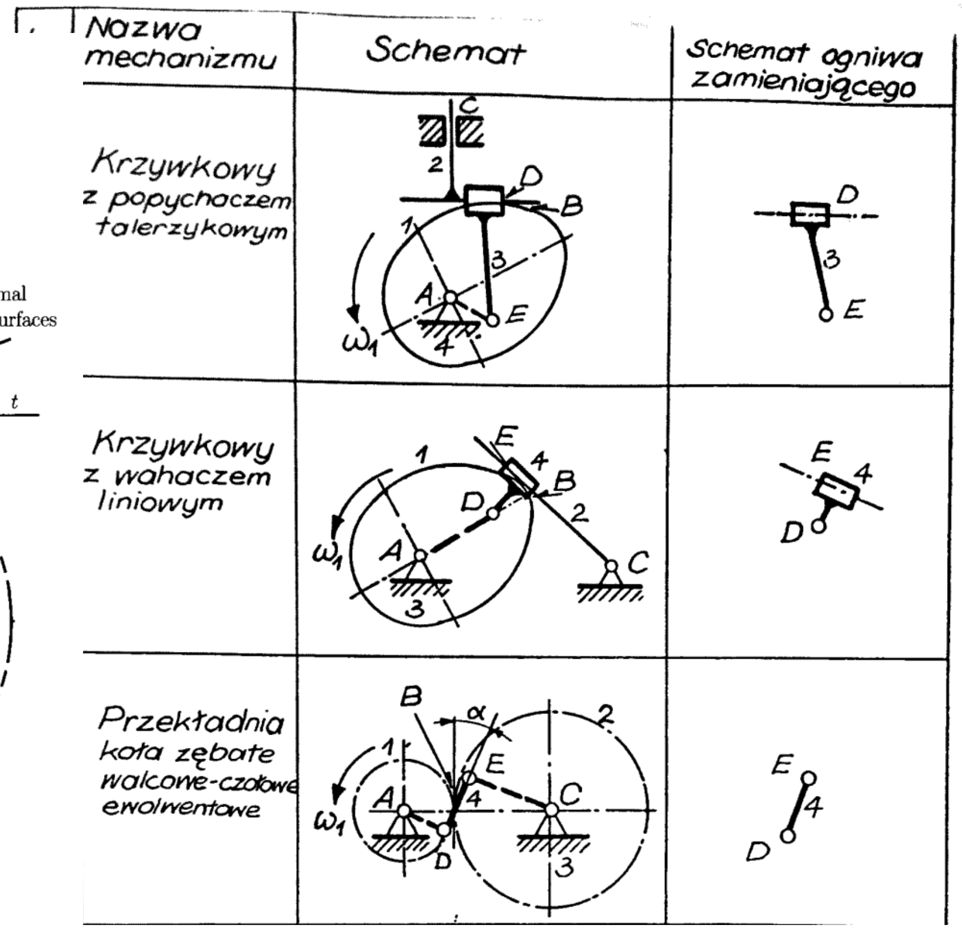


Fig. [Młynarski 1997]

Linkage mechanisms

Examples

Four bar linkage (parallelogram) used in the transmission of wheel drive

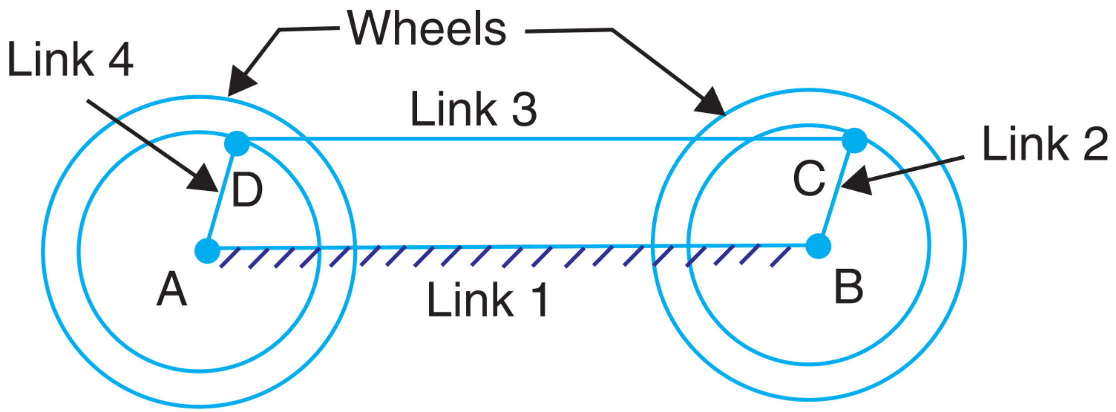


Fig. [Khurmi 2005]

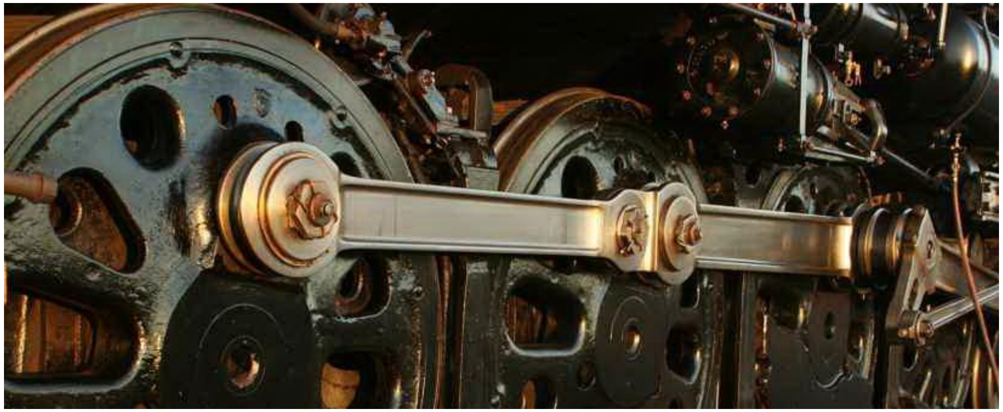


Fig. [https://en.wikipedia.org/wiki/Coupling_rod]

Linkage mechanisms

Examples

Beam engine

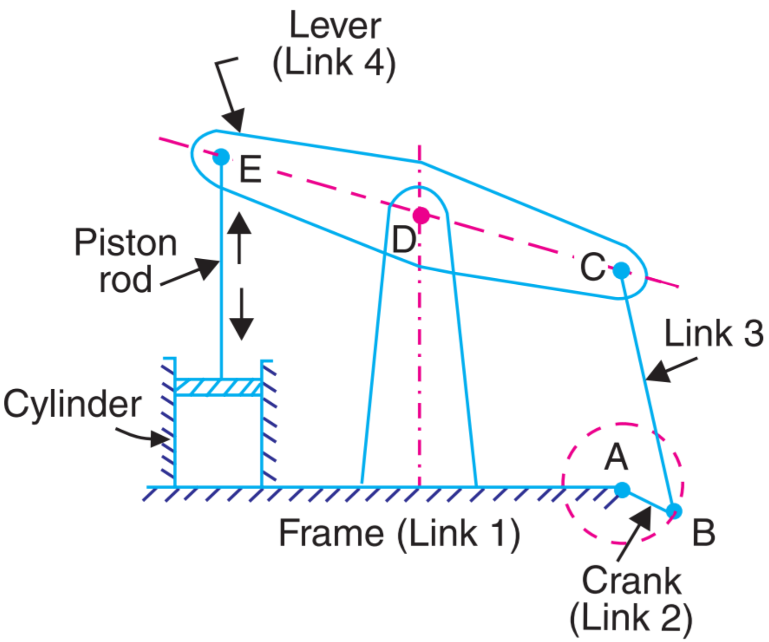


Fig. [Khurmi 2005]

Fig. [https://hanselygretel.eu/tienda/maquina-vapor-watt/]

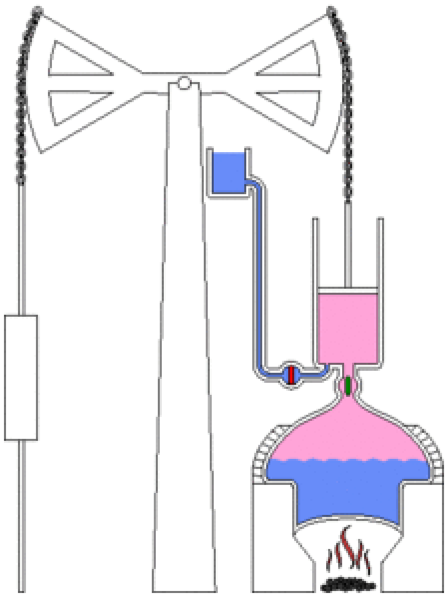


Fig. [https://en.wikipedia.org/wiki/Thomas_Newcomen]

Examples

Slidr crank mechanism

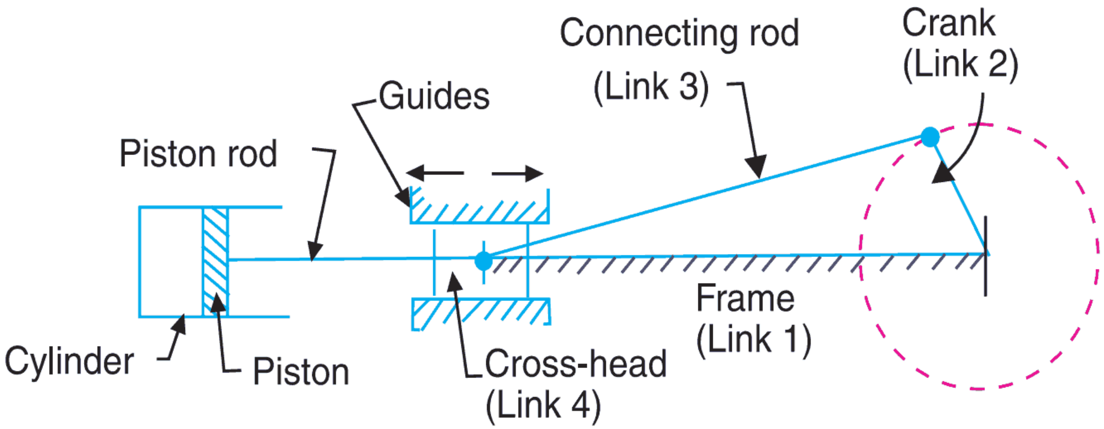


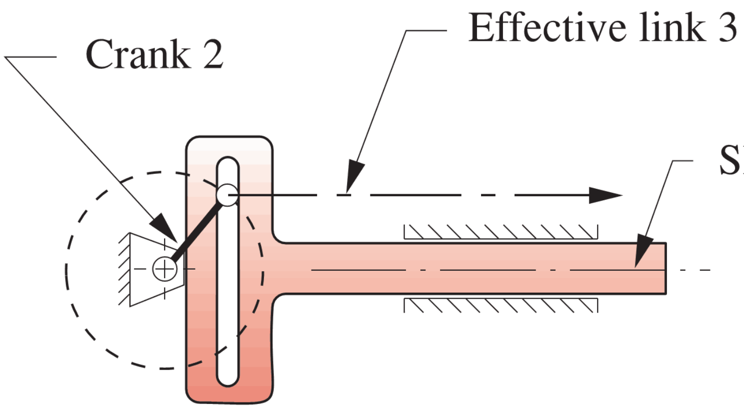
Fig. [Khurmi 2005]

Fig. [http://www.magoda.com/industrial/oem-manufacturing-in-the-united-states/]

Linkage mechanisms

Examples

Scotch yoke mechanisn



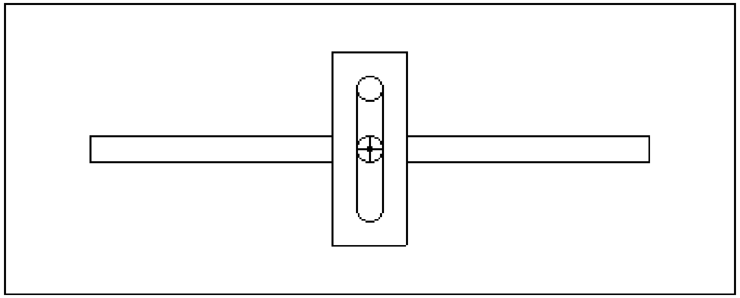
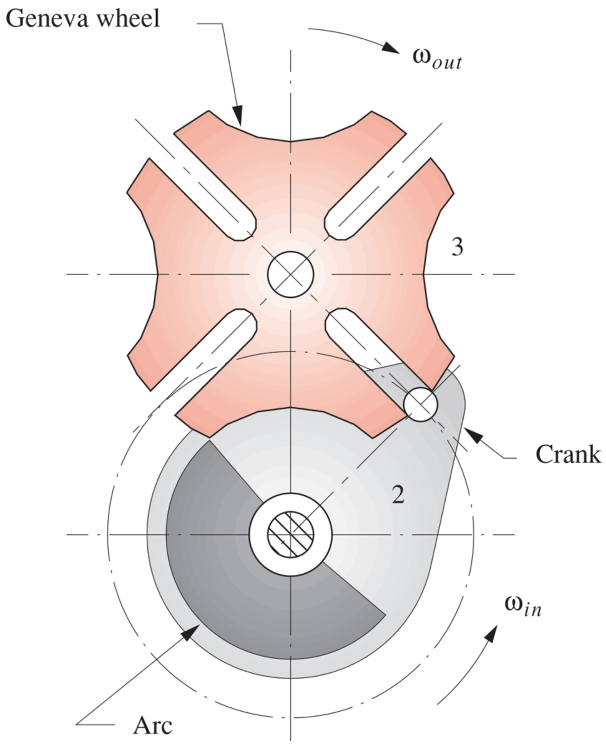


Fig. [https://en.wikipedia.org/wiki/Scotch_yoke#/media/File:Scotch_yoke_animation.gif]

Fig. [Norton 1999]

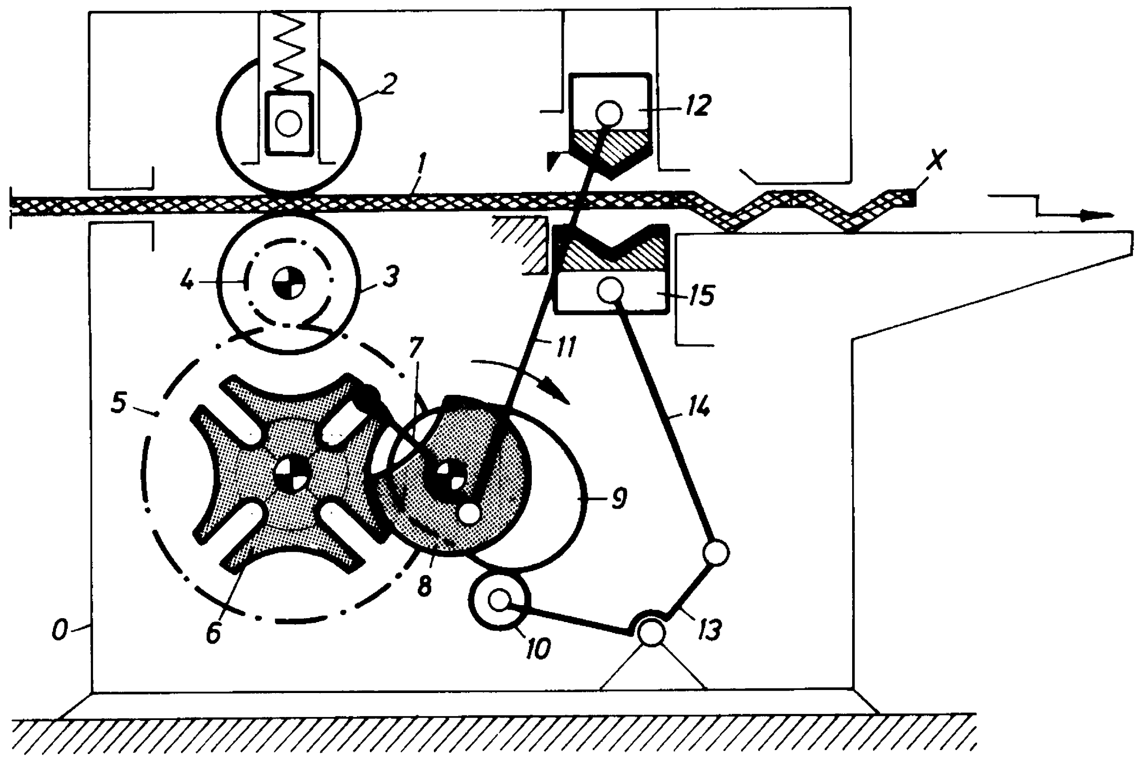
Fig. Scotch yoke mechanism [http://www.etotheipiplusone.net/?m=201209]



Examples

Geneva mechanism

Fig. Geneva mechanism [Norton 1999]



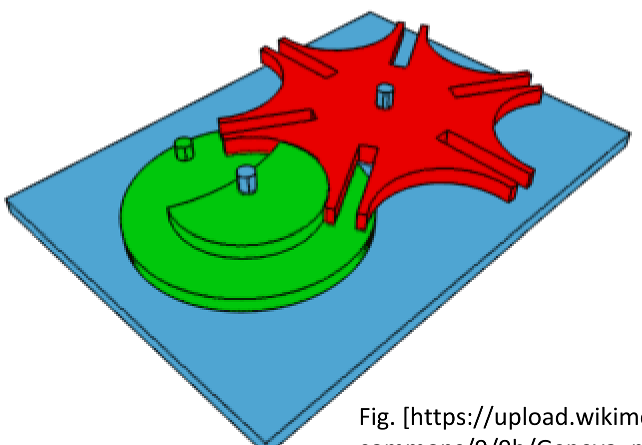


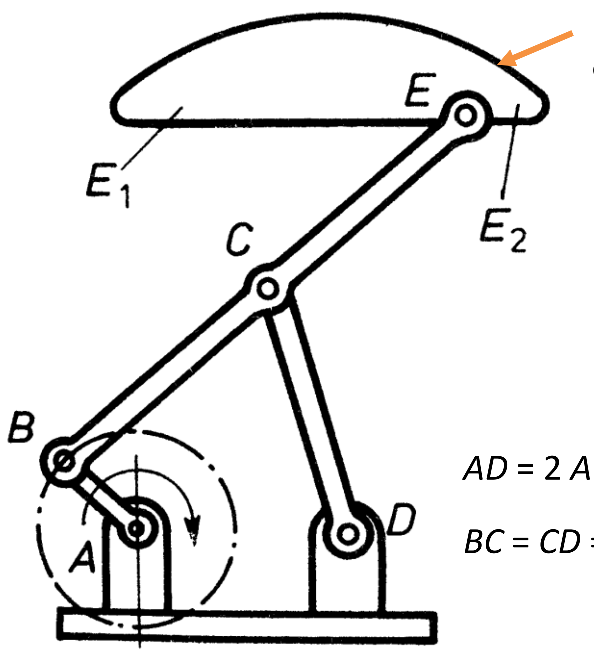
Fig. Sheet metal processing machine
[Miller 1989]

Fig. [https://upload.wikimedia.org/wikipedia/
commons/9/9b/Geneva_mechanism_6spoke_animation.gif]

Examples

Straight-line mechanism

In the 17th century, before the development of milling machines, it was difficult to obtain precise sliding pairs. Movement along a straight line was achieved through the use of relvolute pairs. Straight-lines mechanism are part of steering mechanisms, the task of which is to move along the required trajectory.



$AD = 2\,AB$

$BC = CD = CE = 2{,}5\,AB$

Fig. Chebyschev-Hoekens linkage (approximate straight line) [Morecki 1987]

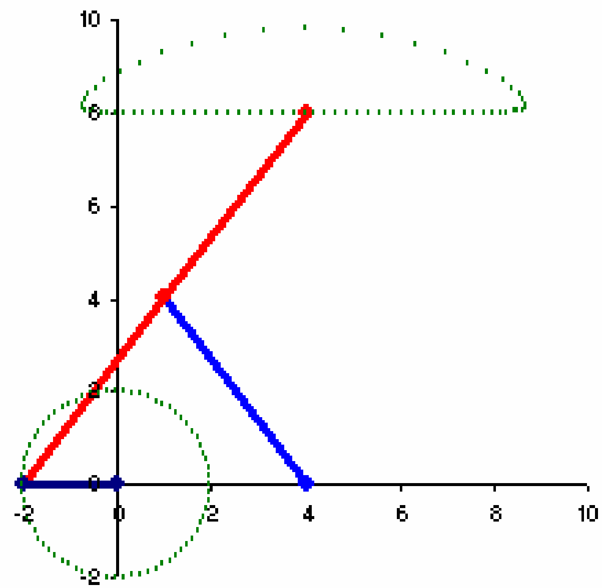


Fig. [https://upload.wikimedia.org/wikipedia /commons/b/be/Hoekens_linkage_animated.gi]

Linkage mechanisms

Examples

Straight-line mechanism

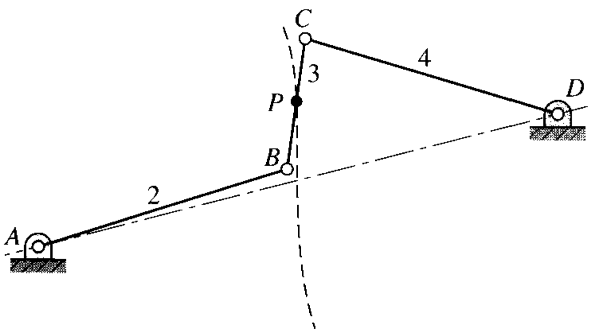


Fig. Wattsa linkage (approximate straight line) [Uicker 2011]

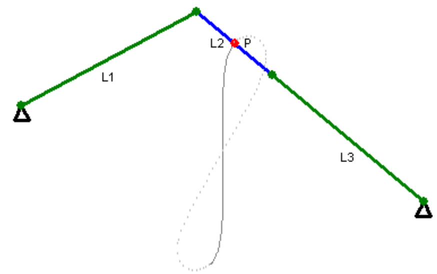


Fig. [https://upload.wikimedia.org/wikipedia/commons/9/93/Watts_linkage.gif]

BC = BP = EC = EP

AB = AE

AD = CD

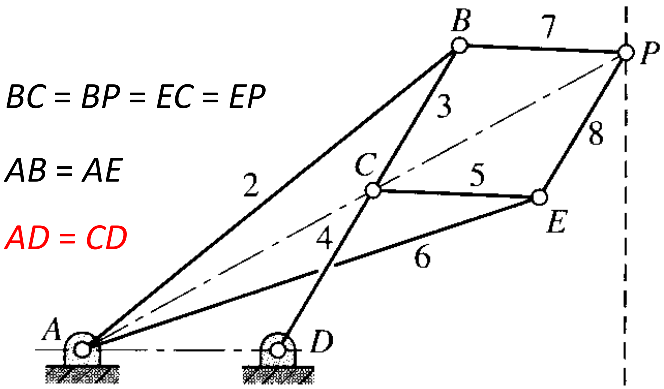


Fig. Peaucellier-Lipkin linkage (exact straight line) [Uicker 2011]

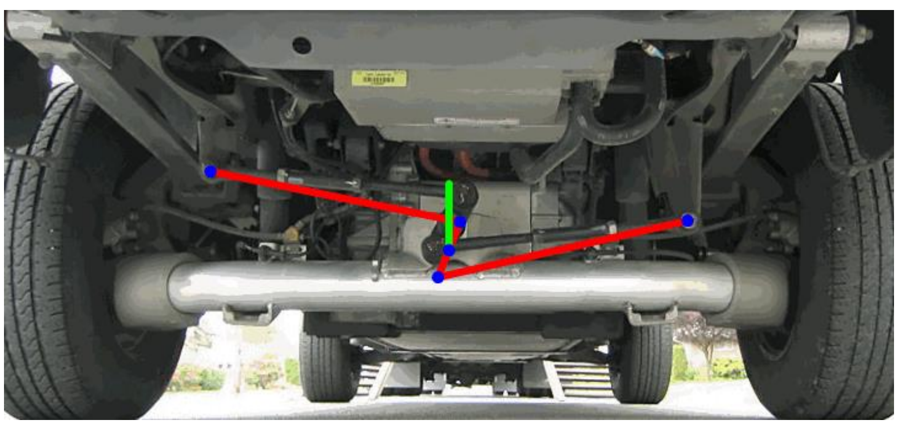


Fig. [https://en.wikipedia.org/wiki/Watt%27s_linkage#/media/File:Watt%27s_Linkage_Rear_Suspension.gif]

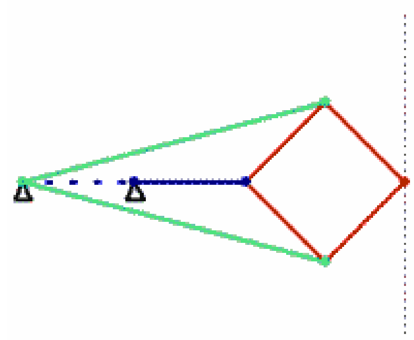


Fig. [https://en.wikipedia.org/wiki/Straight_line_mechanism#/media/File:Peaucellier_linkage_animation.gif]

Linkage mechanisms

Examples

Straight-line mechanism

AD = 4, AB = CD = 5, BC = 2

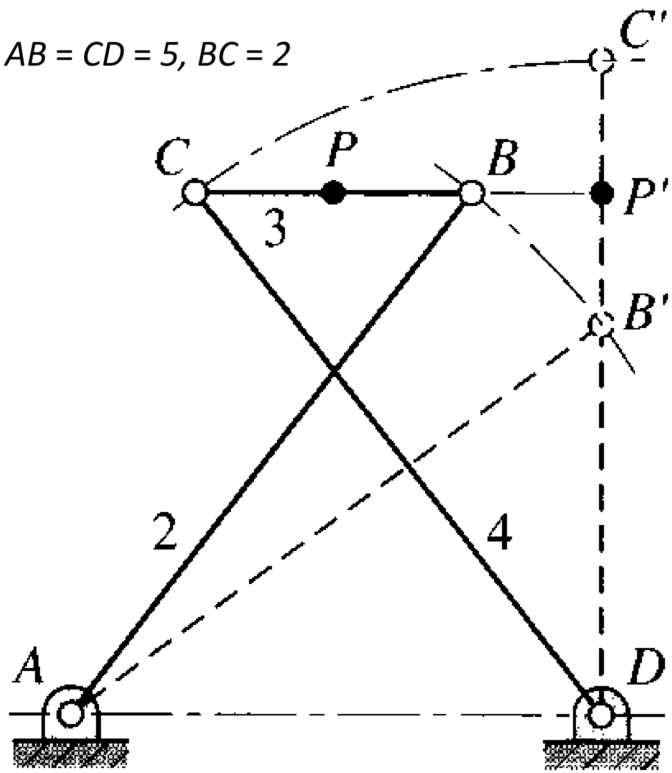


Fig. Chebyschev linkage (approximate straight line) [Uicker 2011]

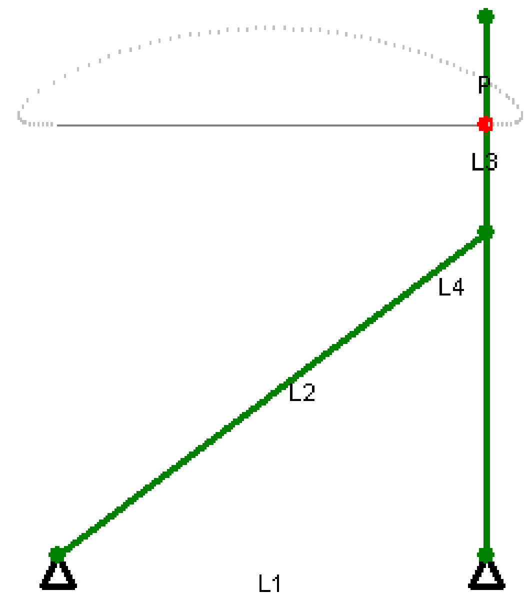


Fig. [https://en.wikipedia.org/wiki/Chebyshev_linkage#/media/File:Chebyshev_linkage.g]if

Linkage mechanisms

Examples

Straight-line mechanism

BC = AD/2

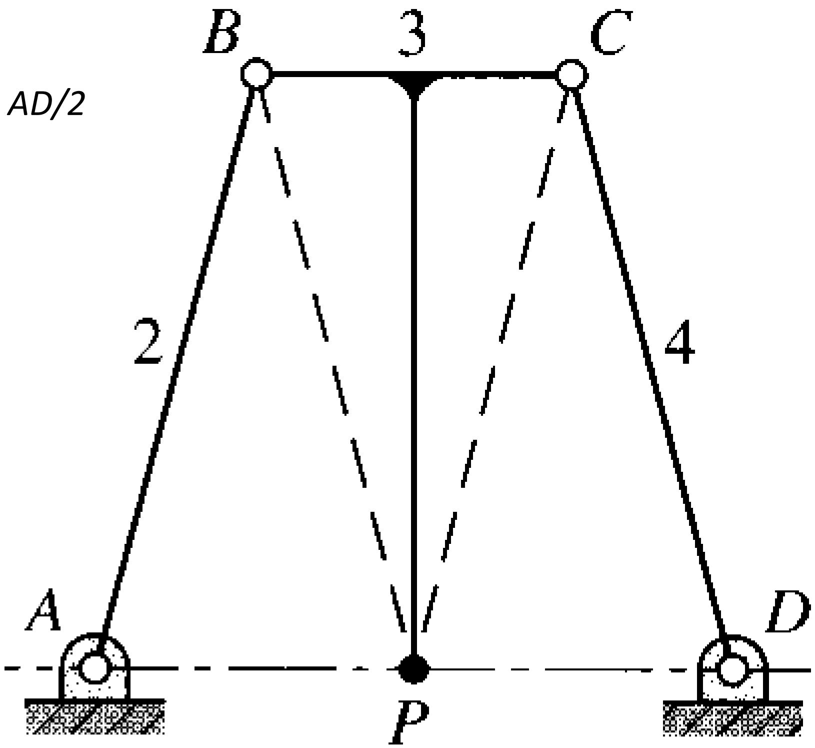


Fig. Roberts linkage (approximate straight line) [Uicker 2011]

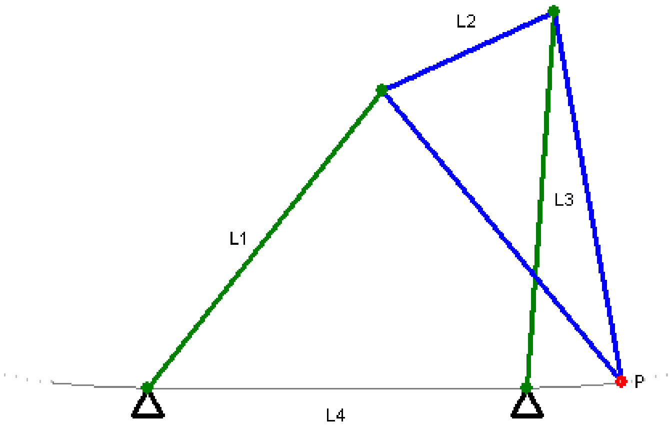


Fig. [https://en.wikipedia.org/wiki/Roberts_Mechanism#/media/File:Roberts_linkage.gif]

Linkage mechanisms

Examples

Straight-line mechanism

Fig. [https://pt.depositphotos.com/2975201/stock-photo-the-port-crane.html]

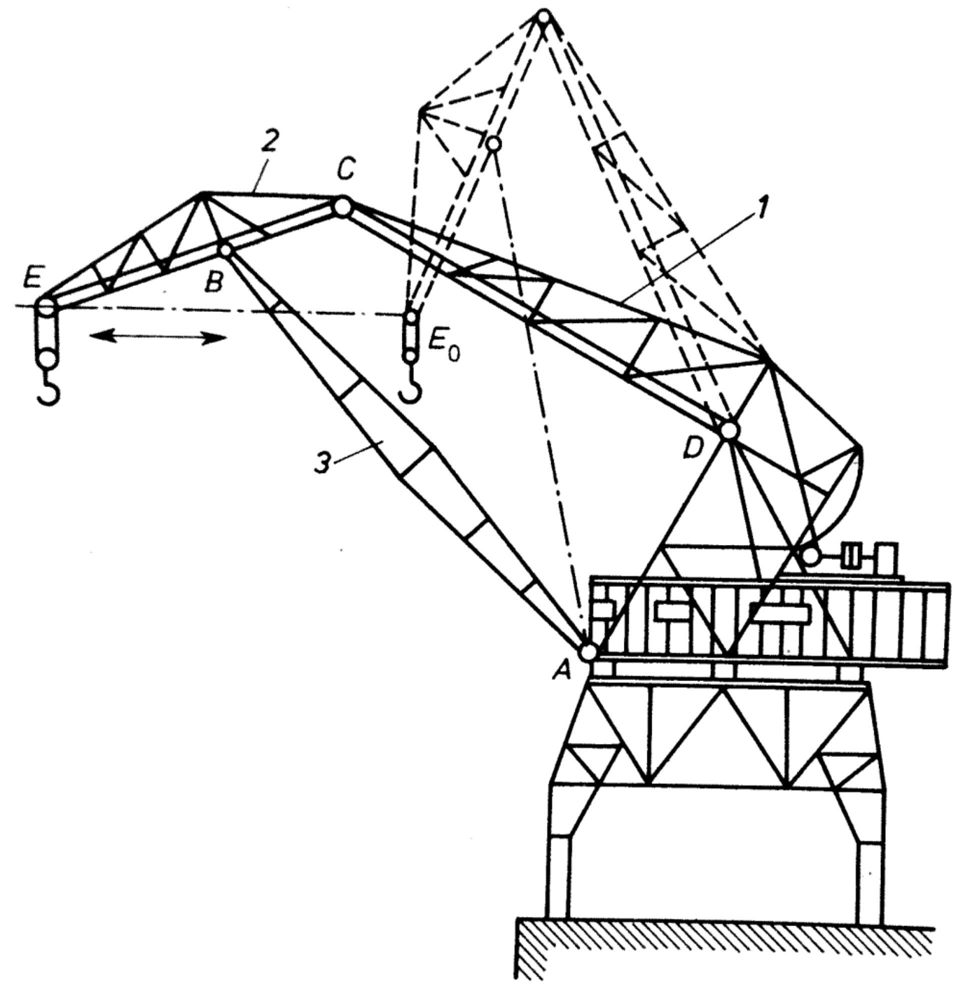


Fig. Port crane, coupler curve EE_o approximate straight line [Morecki 1987]

Examples

Straight-line mechanism

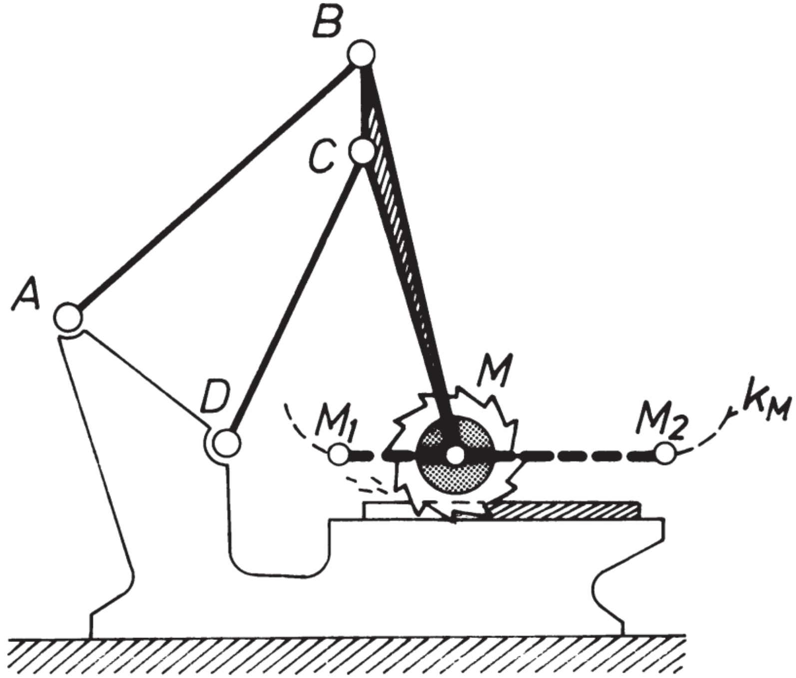


Fig. coupler curve M_1M_2 approximate straight line [Miller 1989]

Examples

Cardana (Hook, universal) joint (variable velocity joint)

Spherical four bar linkage

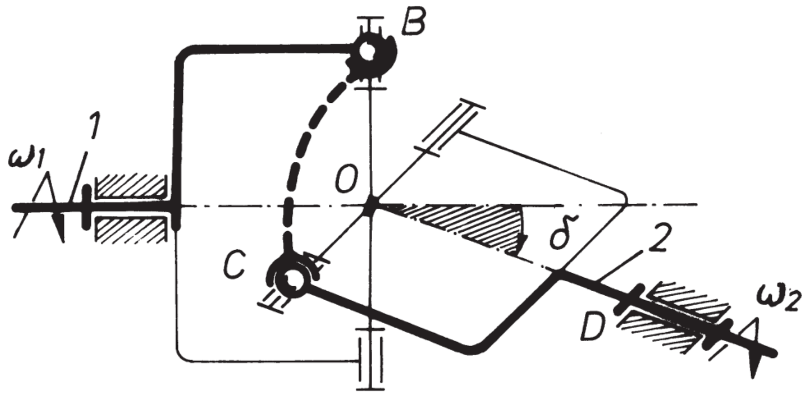


Fig. [Miller 1989]

Fig. Double Cardan joint [agromer.pl/ogolna-wal-kardana-john-deere-al117415,c2,p5717,pl.html]

Linkage mechanisms

Examples

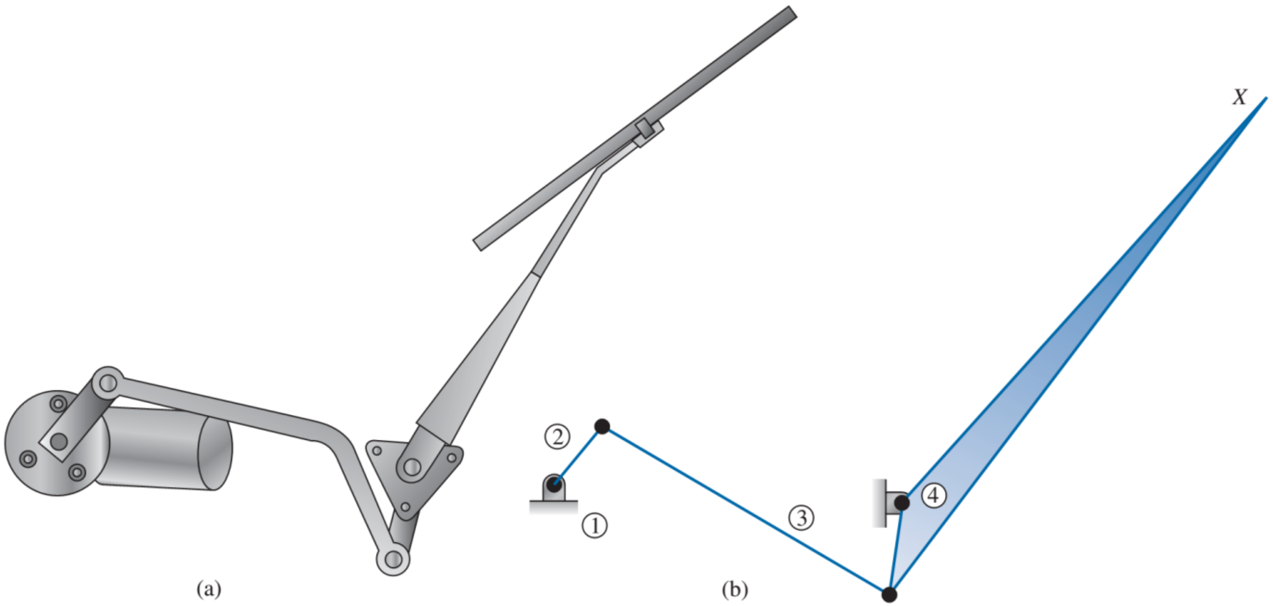


FIGURE 1.33 Rear-window wiper mechanism.

Fig. [Myszka 2012]

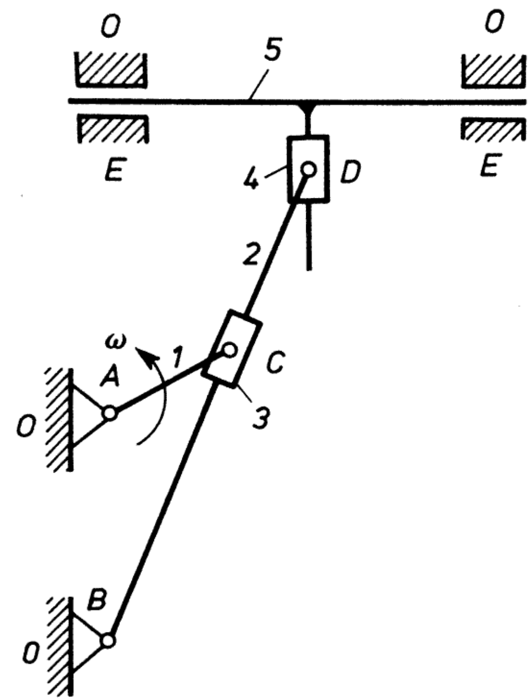


Fig. The drive mechanism of the cross planer table [Morecki 1987]

Literature

1. Gronowicz A.: Podstawy analizy układów kinematycznych. OWPW, Wrocław 2003.
2. Szrek J.: Synteza układu kinematycznego i sterowania czworonożnego robota kołowo-kroczącego. Rozprawa doktorska, Wrocław 2008.
3. Miller S.: Teoria maszyn i mechanizmów. Analiza układów kinematycznych. WPW, Wrocław 1989.
4. Felis J., Jaworowski H., Cieślik J.: Teoria maszyn i mechanizmów. Część I. Analiza mechanizmów. UWND AGH, Kraków 2008.
5. Felis J., Jaworowski H.: Teoria maszyn i mechanizmów. Część II. Przykłady i zadania. UWND AGH, Kraków 2007.
6. Wawrzecki J.: Teoria maszyn i mechanizmów. WPŁ, Łódź 1994.
7. Uicker J., Pennock G., Shigley J.: Theory of machines and mechanism. OUP, New York 2011.
8. Młynarski T., Listwan A., Pazderski E.: Teoria maszyn i mechanizmów. Cz. I. Synteza i analiza strukturalna mechanizmów. ZGPK, Kraków 1997
9. Norton R. L.: Design of machinery. An introduction to the synthesis and analysis of mechanisms and machines. Mc Graw-Hill, 1999.
10. Marghitu D. B.: Mechanisms and robots analysis with Matlab. Springer, 2009.
11. Marghitu D. B.: Kinematic chains and machine components design. Elsevier 2005.
12. Dobrzański T.: Rysunek techniczny maszynowy. WNT, Warszawa 2002.
13. Khurmi R. S., Gupta J.K.: Theory of machines. S Chand & Co Ltd, 2005.
14. Ceccarelli M. (Editor): Distinguished Figures in Mechanism and Machine Science. Their Contributions and Legacies, Part 3. Evgrafov A., Kozlikin D.: Leonid Assur. Springer, Dordrecht 2014.
15. Siemieniako F.: Teoria maszyn i mechanizmów z zadaniami. DzWiPPB, Białystok 1999.
16. Myszka D. H.: Mechines & mechanism. Applied kinematic analysis. Prentice Hall, Boston 2012.
17. Parszewski Z.: Teoria maszyn i mechanizmów. WNT, Warszawa 1978.
18. Morecki A., Oderfeld J.: Teoria maszyn i mechanizmów. PWN, Warszawa 1987.
19. Ambekar A. G.: Mechanism and Machine Theory. Prentice Hall of India, New Delhi 2007.