Lublin University of Technology
Departament of Machine Design and Mechatronics

Theory of Machines

Lecture 1

Basic terminology and structural analysis
Linkage mechanisms

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Basic terminology and definitions

The **theory of machines and mechanisms** is a chosen area of knowledge in the field of engineering and technical sciences. There are two basic tasks posed to TMM - analysis and synthesis. In **analysis**, the main problems to be solved are:

- structure examination,
- kinematic analysis,
- dynamic analysis.

On the other hand, among the objectives of synthesis, one may distinguish the following:

- searching for the structure of mechanisms for given requirements,
- searching for mechanism properties which satisfy the kinematic or dynamic requirements set for them.

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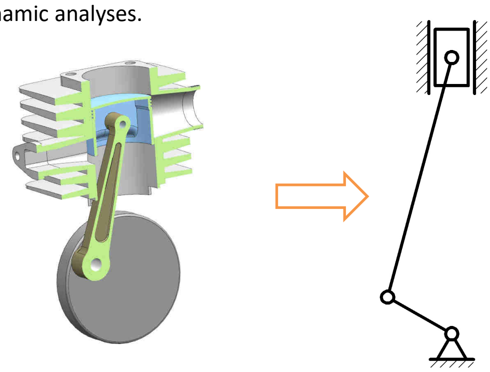
Basic terminology and definitions

In **analysis**, the main problems to be solved are:
- structure examination,
- kinematic analysis,
- dynamic analysis.

The examination of the structure of machines and mechanisms is intended to classify them and determine their mobility. The resulting kinematic diagram is employed in both kinematic and dynamic analyses.



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Basic terminology and definitions

In **analysis**, the main problems to be solved are:
- structure examination,
- kinematic analysis,
- dynamic analysis.

Kinematic analysis seeks to examine motion by determining the displacement, velocity, and acceleration of the components of a mechanism, while disregarding the effects of their mass and any external forces applied.

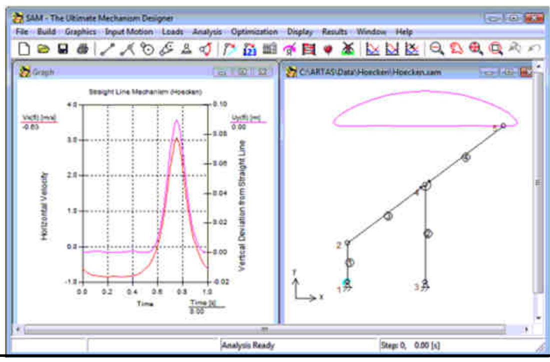

Fig. SAM program (Synthesis and Analysis of Mechanisms) [https://www.artas.nl/en/]

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Basic terminology and definitions

In **analysis**, the main problems to be solved are:

- structure examination,
- kinematic analysis,
- dynamic analysis.

Dynamic analysis concerns the study of mechanisms with respect to the forces acting upon them. The results obtained from kinematic analysis are subsequently utilised.

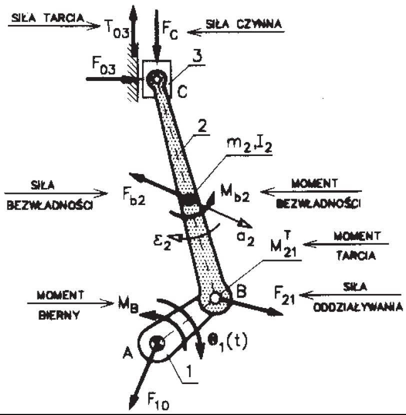

Fig. [Gronowicz 2003]

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Basic terminology and definitions

On the other hand, among the objectives of **synthesis** are:

- searching for the structure of mechanisms for given requirements,
- searching for mechanism properties which satisfy the kinematic or dynamic requirements.

Table 1. Possible variants of the kinematic chain [Szrek 2008]

Number of links k	Joints class I p_1	Joints class II p_2	symbol U_i (k p_1 p_2)
0	0	1	0 0 1
1	2	0	1 2 0
2	3	1	2 3 1
3	5	0	3 5 0

Table 2. Basic structural diagrams [Szrek 2008]

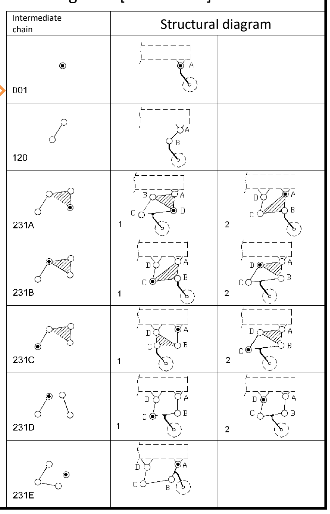
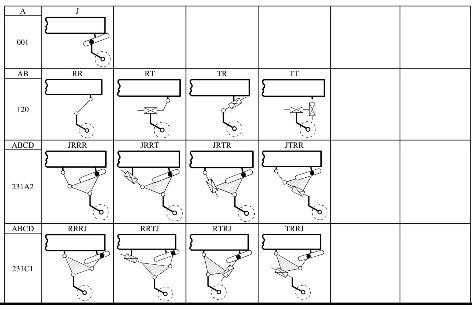


Table 3. Selected kinematic diagrams [Szrek 2008]



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Basic terminology and definitions

On the other hand, among the objectives of **synthesis** are:

- searching for the structure of mechanisms for given requirements,
- searching for mechanism properties which satisfy the kinematic or dynamic requirements.

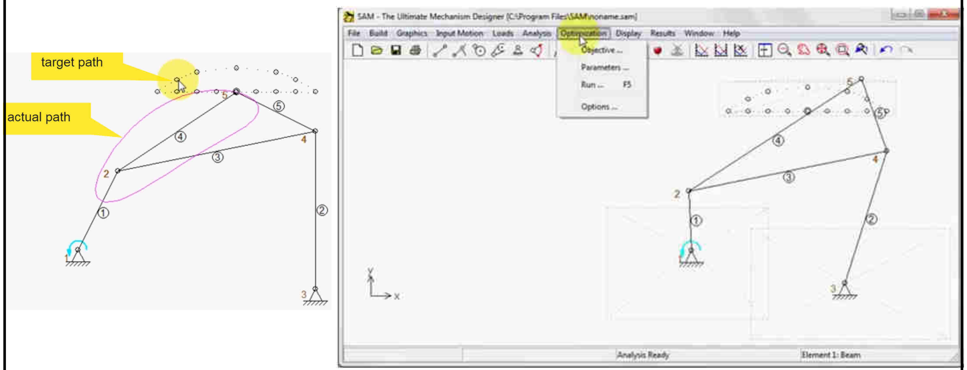

Fig. SAM program [Artas]

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Basic terminology and definitions

Mechanism

1. Selected definitions excludion manipulators as mechanisms:

- Franz Reuleaux (1829-1905) – „assemblage of resistant bodies, connected by movable joints, to form a **closed kinematic chain** with one link fixed and having the purpose of transforming motion” [Uicker 2011].

- Miller S. – a closed kinematic chain with one fixed link characterized by the number of driver links equal to its mobility.

A *closed kinematic chain* is a system of links connected by joints in such a way that each link forms a pair with at least two other links.

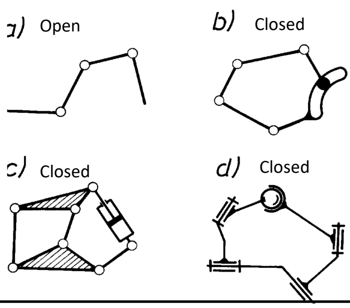

Fig. Kinematic chains [Miller 1989]

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Basic terminology and definitions

Mechanism

2. Selected definitions treating manipulators as mechanisms :

- Felis, Jaworowski, Cieślik – is a kinematic chain performing a precisely defined motion.

A kinematic chain is a set of links connected in kinematic pairs, with one link fixed.

- Wawrzecki J. – *is a set of connected links with precisely defined relative motion, whose function is to transmit motion*.

There are also definitions that exclude natural creations as mechanisms.

Spatial mechanism

Every real mechanism is a spatial mechanism. Planar mechanisms are distinguished from this group to facilitate simpler kinematic, and partly structural and dynamic, analysis.

A ***planar mechanism*** is one in which all points of all members move in planes that are mutually parallel to a stationary plane.

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Basic terminology and definitions

Machine

- Franz Reuleaux (1829-1905) – „combination of resistant bodies so arranged that by their means the mechanical forces of nature can be compelled to do work accompanied by certain determinate motions" [Uicker 2011].

- Felis, Jaworowski, Cieślik – a set of mechanisms that perform the desired work related to technological processes or energy transformation.

- Miller S. – is *a device in which mechanical motion is involved in a power process that results in useful work or energy transformation*.

What is the difference between a machine and a mechanism?

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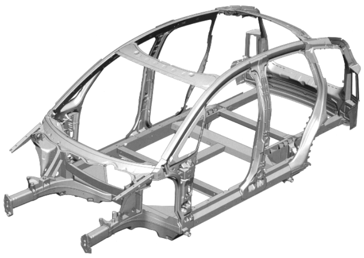
Basic terminology and definitions

Structure – rigid bodies connected in such a way that relative motion is not possible and designed to carry loads.



Is this an area of interest for TMM?

Fig. Vehicle frame
[https://mechanics.stackexchange.com/tags/frame/info]

Fig. Bridge
[qctimes.com/traffic/photos-new-i--bridge/collection_6fedb8c6-c1ae-11e3-a1b9-0019bb2963f4.html]

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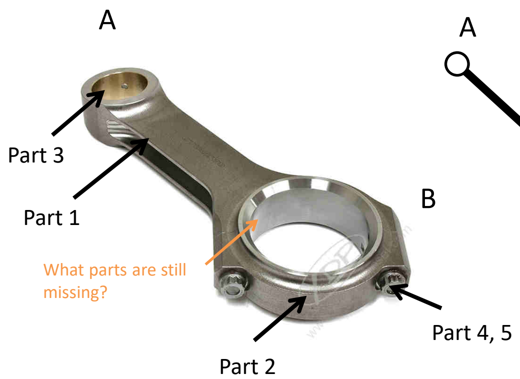
Basic terminology and definitions

Link (**member**):
an element, or a set of rigidly connected elements, of which a mechanism or machine is composed, that mediates the transmission of motion.

Connecting rod



Connecting rod – symbolic representation

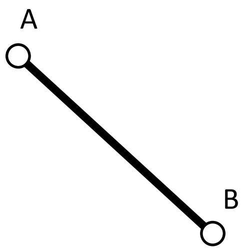


Fig. [http://www.xtremediesel.com/Carrillo-Cummins-Pro-H-Connecting-Rod-with-H-11-Bolts.aspx]

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Basic terminology and definitions

Types of links (members)

1. Classification by the motion performed relative to the adopted reference system:

- movable (e.g. shaft, piston, rod) - having motion relative to an assumed reference system,
- fixed (e.g. base, frame, body) - having no motion relative to the reference system.

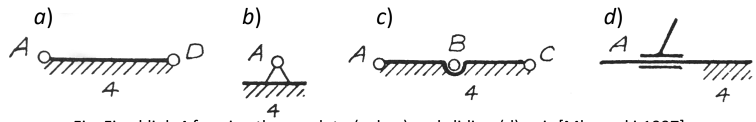

Fig. Fixed link 4 forming the revolute (a, b, c) and sliding (d) pair [Młynarski 1997]

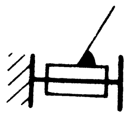
Fig. A fixed link that is part of a spatial revolute pair [Dobrzański 2002]

Basic terminology and definitions

Types of links (members)

2. Classification by the performed role:

- driver (driving) - whose motion is known,
- passive (driven),
- mediating.

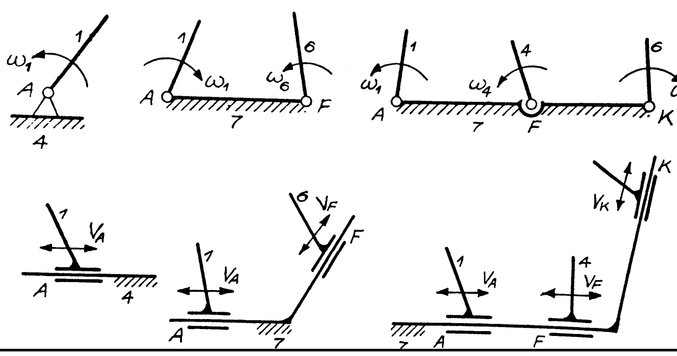

Fig. Driver link marked with an arrow [Młynarski 1997].

Basic terminology and definitions

Types of links (members)

3. Classification by susceptibility:
- rigid - assumed to be non-deformable,
- susceptible, e.g., a spring,
- liquids and gases.

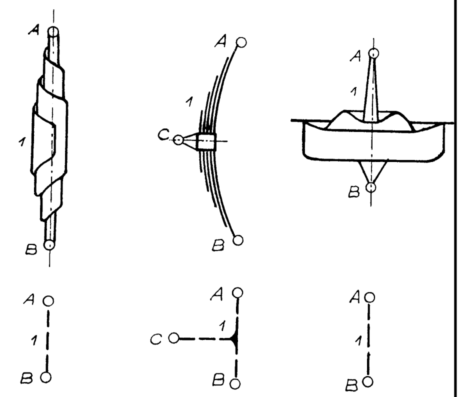

Fig. Susceptible link [Młynarski 1997]

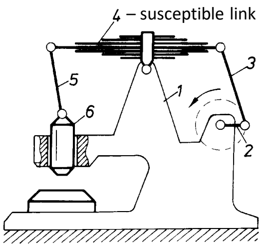

Fig. [Miller 1989]

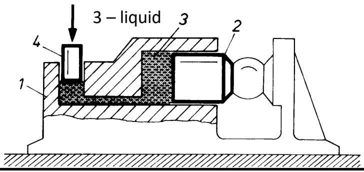

Fig. [Miller 1989]

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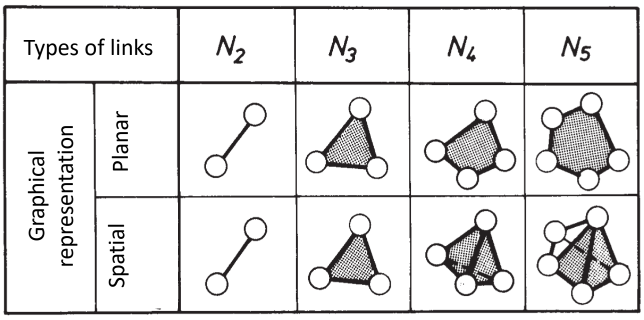
Basic terminology and definitions

Types of links (members)
According to numer of nodes



Binary link Ternary link Quaternary link Quintuple link

Fig. [Miller 1989]

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Basic terminology and definitions

Joint (kinematic pair, pair) – a connection between two or more links that allows relative motion between them while imposing certain physical constraints on that motion.

Classification of joints according to:
1. Type of relative motion between the elements.
2. Type of contact between the elements.
3. Type of closure between the elements.
4. Number of links joined.
5. Number of degrees of freedom in the relative motion.

Basic terminology and definitions

Classification of joints according to:
1. Type of relative motion between the elements.

- ***a)*** ***Revolute*** **(*Turning*)** ***pair****.* When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link. 1 DOF.
- ***b)*** ***Sliding*** **(*Prismatic, Prism*)** ***pair****.* When the two elements of a pair are connected in such a way that one can only slide relative to the other, the pair is known as a sliding pair. 1 DOF.
- ***c)*** ***Cylindrical pair.*** Permits angular rotation and independent translation motion. Thus, a cylindrical pair has two degrees of freedom. 2 DOF.
- ***d)*** ***Screw*** **(*helical*)** ***pair****.* When the two elements of a pair are connected in such a way that one element can turn about the other by screw threads, the pair is known as screw pair. The lead screw of a lathe with nut and bolt with a nut are examples of a screw pair. 1 DOF.
- ***e)*** ***Spherical pair.*** When the two elements of a pair are connected in such a way that one element (with spherical shape) turns or swivels about the other fixed element, the pair formed is called a spherical pair. The ball and socket joint, attachment of a car mirror, pen stand etc., are the examples of a spherical pair. 3 DOF.
- ***f)*** ***Planar pair*** **(*flat*).** It is seldom found in mechanism in its undisguised form, except at a support point. It has three degrees of freedom, that in, two translations and a rotation. 3DOF.

Basic terminology and definitions

Classification of joints according to the :
1. Type of relative motion between the elements.

Theoretically possible:

g) Spherical pair with pivot. The links are connected in such a way that they can perform rotational movements relative to each other about two axes. 2 DOF.

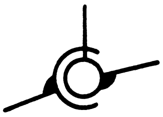
Fig. [Dobrzański 2002]

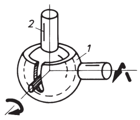

Fig. [Miller 1989]

h) Sphere-cylinder pair. The links are connected in such a way that they can perform rotary motion relative to each other about three axes and rectilinear motion relative to one axis. 4 DOF.

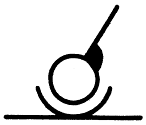
Fig. [Dobrzański 2002]

i) Sphere-plane pair. The legs are connected in such a way that they can perform rotational motion about three axes and translational motion about two axes with respect to each other. 5 DOF.

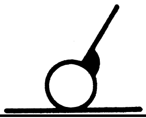
Fig. [Dobrzański 2002]

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Basic terminology and definitions

Schematic representation	Construction	Technical realisation

a) Revolute (Turning) pair – I

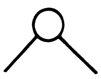
Fig. Plane
[Dobrzański 2002]

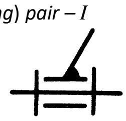
Fig. Spatial
[Dobrzański 2002]

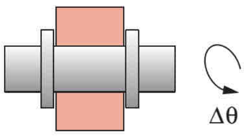

Fig. [Norton 1999]

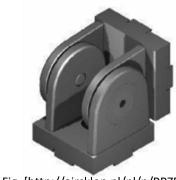
Fig. [http://airsklep.pl/pl/p/PRZEGUB-OBROTOWY-BEZ-DZWIGNI-30x30-B/181]

b) Sliding (Prismatic, Prism) pair- I

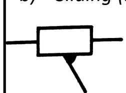
Fig. Plane
[Dobrzański 2002]

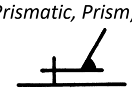
Fig. Spatial
[Dobrzański 2002]

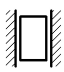
Fig. Plane

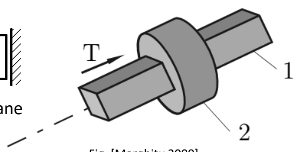

Fig. [Marghitu 2009]


Fig. [http://www.gg-powertransmission.com/splined-shafts-sleeves/standard/]

c) Cylindrical pair - II

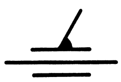
Fig. [Dobrzański 2002]

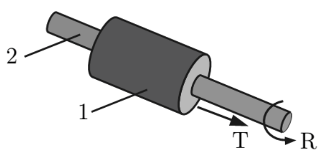

Fig. [Marghitu 2009]


Fig. [http://www.practicalmachinist.com/vb/archive/index.php/t-274723.html]

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Basic terminology and definitions

Schematic representation | **Construction** | **Technical realisation**

d) Screw pair – I

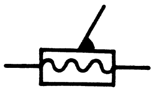
Fig. Version 1 [Dobrzański 2002]


Fig. Version 2 [Dobrzański 2002]

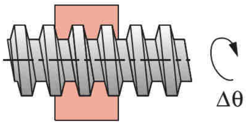

Fig. [Norton 1999]

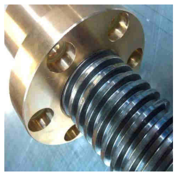
Fig. [http://www.linearways.in/lead-screws-nuts.html]

e) Spherical pair – III


Fig. [Dobrzański 2002]

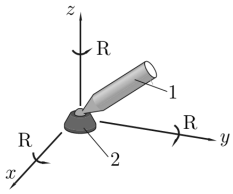

Fig. [Marghitu 2009]


Fig. [https://en.wikipedia.org/wiki/Ball_joint]

f) Flat (planar) pair – III

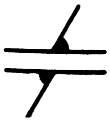
Fig. [Dobrzański 2002]

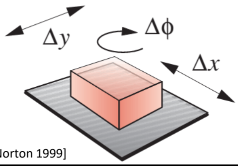

Fig. [Norton 1999]


Fig. [http://sofy24.pl/sofy/162-kanapa-3-osobowa-rozkladana-dawid.html]

Basic terminology and definitions

2. *The kinematic pairs according to the type of contact between the elements may be classified as discussed below* (*Reuleaux*)*:*

a) Lower pair. When the two elements of a pair have surface contact during relative motion, and the surface of one element slides over the surface of the other, the pair formed is known as alower pair. Examples of lower pairs (in technical realizations) include: revolute, sliding, screw, cylindrical, sphere and planar.

b) Higher pair. When the two elements of a pair have (theoretically) a line or point contact during relative motion, and the movement between the two elements involves both turning and sliding, the pair is known as a higher pair. Examples of higher pairs having 2 DOF include: gear (half) joint (gear contact) and cam-follower (half) joint (rolling-sliding contact).


Fig. Gear wheels [http://science.howstuffworks.com/transport/engines-equipment/gear2.htm]

Fig. Number of degrees of freedom, schematic representation and symbolic representation of gear joint [Gronowicz 2003]


Fig. Cam and follower [https://www.cgtrader.com/free-3d-models/industrial/part/cam-follower-mechanism-with-spring]

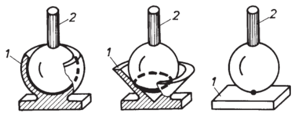

Fig. Theoretical constructions of spherical pair [Miller 1989]

Fig. Number of degrees of freedom and schematic representation of cam and follower [Gronowicz 2003]

Basic terminology and definitions

3. *The kinematic pairs according to the type of closure between the elements may be classified as discussed below :*

a) *Self closed pair.* When the two elements of a pair are connected mechanically in such a way that only the required type of relative motion occurs, the pair is known as a self closed pair.
b) *Force – closed pair.* When the two elements of a pair are not connected mechanically but are kept in contact by the action of external forces, the pair is known as a force-closed pair. The cam and follower in an example of force closed pair, as it is kept in contact by the force exerted by spring and gravity.

Fig. Cam and follower - forced closed
[https://www.cgtrader.com/free-3d-models/industrial/part/cam-follower-mechanism-with-spring]

Fig. Spherical pair – self closed
[https://en.wikipedia.org/wiki/Ball_joint]

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Basic terminology and definitions

4. Classification of joints according to the number of links joined.

The order of a joint is defined as the number of links joined together minuse one. The combination of two links has an order of one and constitutes a single joint.

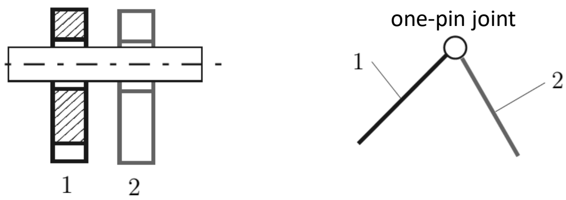

Fig. Joint of order one
[Marghitu 2009]

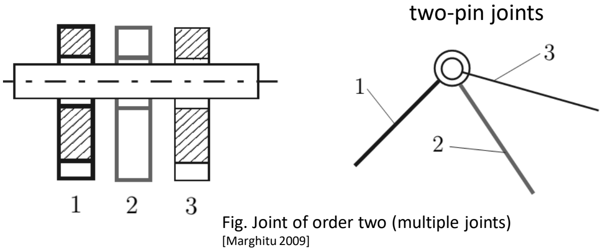

Fig. Joint of order two (multiple joints)
[Marghitu 2009]

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Basic terminology and definitions

5. Classification of joints according to the number of degrees of freedom in relative motion.

This is the most common classification of kinematic pairs. The class of a pair is defined as the number* of left degrees of freedom. A revolute and sliding pair is therefore class I, a cylindrical pair class II, a spherical pair class III, etc.

There also exist yoke joint (pin in slot) in mechanisms, which possesses 2 DOF and has not been previously presented.

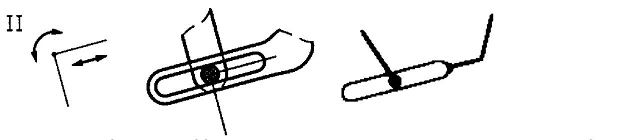

Fig. Number of degrees of freedom, schematic and symbolic representation of the yoke joint [Gronowicz 2003]

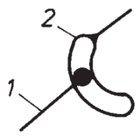

Fig. [Miller 1989]

*Another approach to determining the class of kinematic pairs is also used. In this method, the number of removed degrees of freedom determines the class of the pair. For example, a revolute pair is classified as class V.

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Basic terminology and definitions

Number of degrees of freedom (DOF)

Felis, Jaworowski, Cieślik – *The number of degrees of freedom of a rigid body is the number of independent generalized coordinates that determine its position in space relative to a chosen coordinate system*.

The motion of a free rigid body in space can be represented as three independent translational motions along the X, Y and Z axes and three independent rotational motions about the X, Y and Z axes.

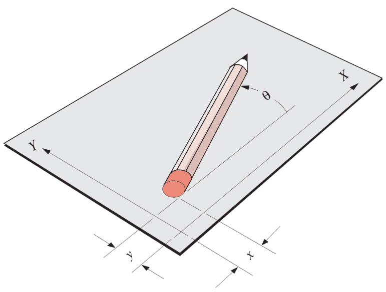

Fig. Rigid body in 2D space (plane) - 3 DOF [Norton 1999]

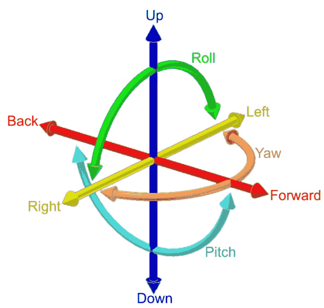

Fig. Rigid body in 3D space - 6 DOF [https://en.wikipedia.org/wiki/Six_degrees_of_freedom]

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Basic terminology and definitions

Number of degrees of freedom (DOF)

Other approaches to understand DOF

- The number of inputs that must be provided to produce a predictable output;
- Number of independent coordinates required to define the position of an element.

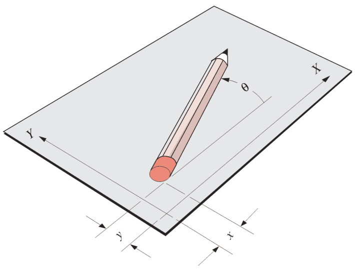

Fig. Rigid body in 2D space (plane) - 3 DOF [Norton 1999]

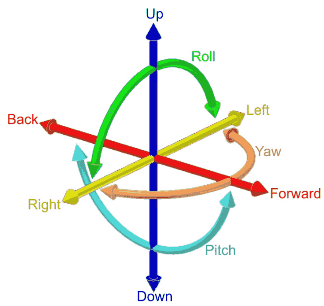

Fig. Rigid body in 3D space - 6 DOF [https://en.wikipedia.org/wiki/Six_degrees_of_freedom]

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Basic terminology and definitions

Mobility *M* (degrees of freedom) of a mechanism

The mobility of a mechanism is the number of degrees of freedom that the links have relative to the base (fixed link).
In other words, the mobility of a mechanism can be defined as the number of drives that must be applied for its motion to be determined.

The formula for calculating the mobility of planar mechanisms, known as the Kutzbach criterion, is as follows:

$$M = 3(n-1) - 2\,p_1 - p_2$$

M = mobility,
n = number of links including frame,
p_1 = number of joints of class I (with 1 DOF),
p_2 = number of joints of class II (with 2 DOF).

and for spatial mechanisms :

$$M = 6(n-1) - 5\,p_1 - 4\,p_2 - 3\,p_3 - 2\,p_4 - p_5$$

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Basic terminology and definitions

Mobility of planar mechanisms - examples

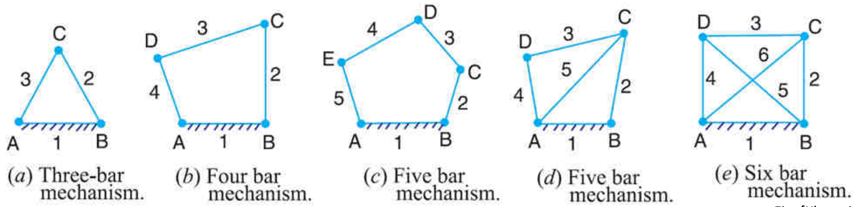

Fig. [Khurmi 2005]

The mobility (M) for some simple mechanisms that do not include higher pairs (i.e. $p_4 = 0$) as shown in figure, is determined as follows:

a) The mechanism has three links and three binary joints i.e. $n = 3$ and $p_5 = 3$

$$M = 3\,(3-1) - 2 \times 3 = 0$$

b) The mechanism has four links and four binary joints i.e. $n = 4$ and $p_5 = 4$

$$M = 3\,(4-1) - 2 \times 4 = 1$$

c) The mechanism has five links and five binary joints i.e. $n = 5$ and $p_5 = 5$

$$M = 3\,(5-1) - 2 \times 5 = 2$$

d) The mechanism has five links and four binary joints (because there are two joints at *B* and *D*, and two ternary joints at *A* and *C*) i.e. $n = 5$ and $p_5 = 6$

$$M = 3\,(5-1) - 2 \times 6 = 0$$

d) The mechanism has six links and eight binary joints (because there are four ternary joints at *A*, *B*, *C* and *D*) i.e. $n = 6$ and $p_5 = 8$

$$M = 3\,(6-1) - 2 \times 8 = -1$$

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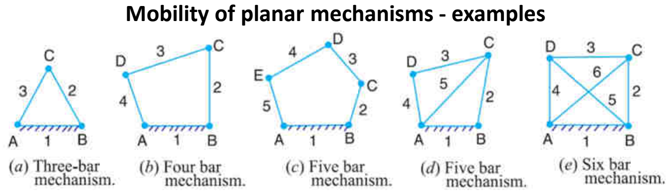
Basic terminology and definitions

Mobility of planar mechanisms - examples

Rys. [Khurmi 2005]



It may be noted that:

- When $M = 0$, then the mechanism forms a structure and no relative motion between the links is possible Fig. *a* and *d*;
- When $M = 1$, the mechanism can be driven by a single input motion, as shown in Fig. *b*;
- When $M = 2$, two separate input motions are required to produce unambiguous motion for the mechanism, as shown in Fig. *c*;
- When $M = -1$ or less, there are redundant constrains in the chain and it forms a statically indeterminate structure, as shown in Fig. *e*.

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Basic terminology and definitions

$M \leq 0$ Mechanism with zero, or negative, mobility are termed locked mechanisms. These mechanisms are unable to move and form a structure.

$M > 1$ A mechanism with multiple mobility requires more than one driver to operate it precisely. In general, this type of mechanism offer greater ability to position a link accurately. There is no requirement that a mechanism have only $M = 1$, although that is often desirable for simplicity of drive control.

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Basic terminology and definitions

Kutzbach's–Gruebler's formula for mechanism mobility does not take into account the specific geometry (size or shape) of the links, but only the connectivity of the links and the type of joints (connections). The **Grübler equation does not always yiels accurate results.**

Redundand DOF (idle DOF)

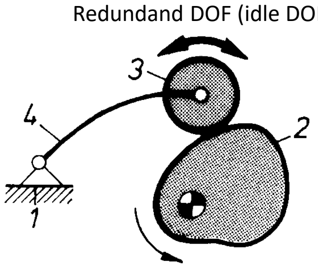
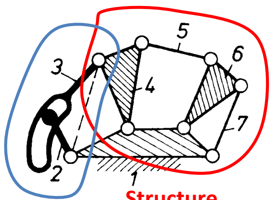


Fig. [Miller 1989]

$M_{real} = 1$
$M_{equation} = 3 \times 3 - 2 \times 3 - 1 = 2$

Incomplete mobility

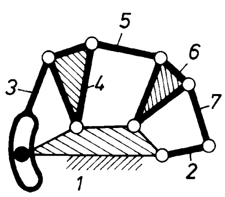


Mechanism
$M_{real} = 1$

Structure
$M_{real} = 0$

$M_{equation} = 3 \times 6 - 2 \times 8 - 1 = 1$

Complete mobility



$M_{real} = 1$
$M_{equation} =$
$3 \times 6 - 2 \times 8 - 1 = 1$

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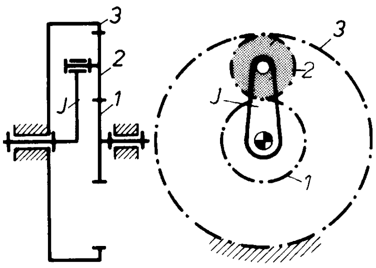
Basic terminology and definitions

Overconstrained mechanism



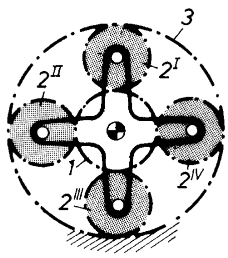


Fig. [Miller 1989]

$M_{real} = 1$
$M_{equation} = 3 \times 3 - 2 \times 3 - 2 = 1$

$M_{real} = 1$
$M_{equation} = 3 \times 6 - 2 \times 6 - 8 = -2$

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Basic terminology and definitions

Overconstrained mechanism

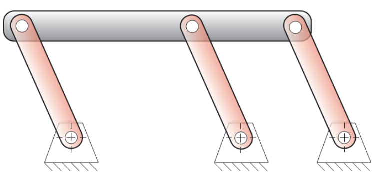

Fig. [Norton 1999]

$M_{real} = 1$
$M_{equation} = 3 \times 4 - 2 \times 6 = 0$

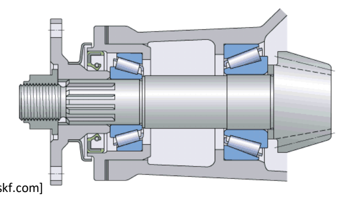

Fig. [www.skf.com]

$W_{real} = 1$
$M_{equation} = 3 \times 1 - 2 \times 2 = -1$

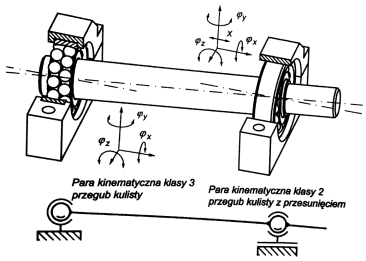

Fig. [Felis 2007]

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Classification of mechanisms

There are three types of mechanism classifications:
- structural,
- constructional,
- functional.

Structural classification, that is, the division according to structural features, was initiated by the Russian scientist Leonid Vladimirovich Assur (1878-1920) and later developed by other researchers. Ivan Ivanovich Artobolevski (1905-1977) further specified the concept of structural group, including class and order [Ceccarelli 2014].

The structural classification adopted in this study divides mechanisms into families, classes, and orders (Fig.). This classification does not allow for the unambiguous assignment of all mechanisms to appropriate categories. More accurate methods of classification have been developed, with a greater degree of complexity, however, the discussion of these methods is beyond the scope of this presentation.

The advantage of structural classification lies in its ability to facilitate the development of universal methods for structural, kinematic, and dynamic analysis.

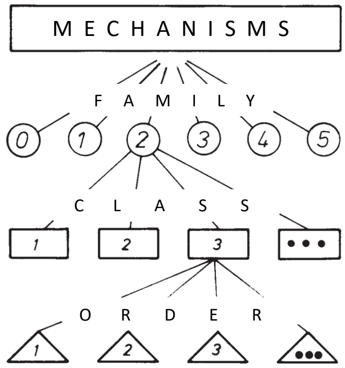

Fig. [Miller 1989]

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Classification of mechanisms - structural

Family

There are six families numbered from 0 to 5. Membership in a family is determined by the imposed constraints. They are defined relative to the adopted coordinate system jointly for all members. In **family 0**, the links possess a total of six degrees of freedom (Figure 1)

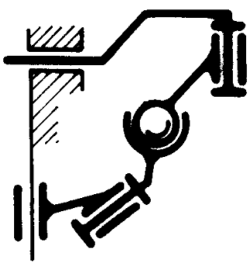

Fig. 1. Family 0 – 6 DOF in total [Miller 1989]

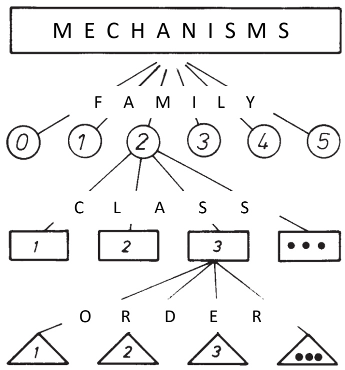

Fig. [Miller 1989]

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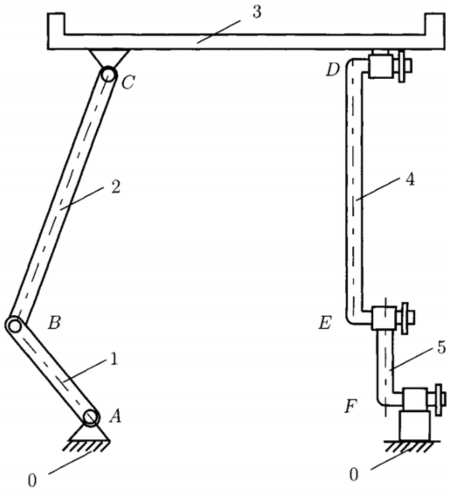
Classification of mechanisms - structural

Family

Family 1 – all links do not have the same DOF.



Link	T_x	T_y	T_z	R_x	R_y	R_z
0	No	No	No	No	No	No
1	No	No	No	No	No	Yes
2	Yes	Yes	No	No	No	Yes
3	No	Yes	No	No	No	No
4	No	Yes	Yes	Yes	No	No
5	No	No	No	Yes	No	No
					No	

For all links R_y = No $\Longrightarrow f = 1$.

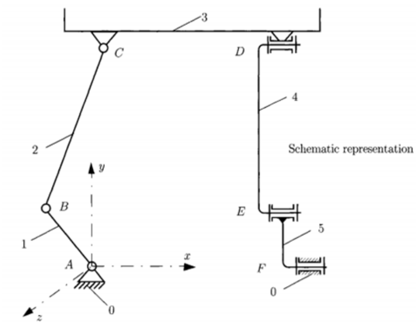


Fig. Family 1 – 5 DOF in total [Marghitu 2005]

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Classification of mechanisms - structural

Family

Family 2 – all links have the same degrees of freedom removed. For the mechanism in Figure 2, these are rotation about the vertical axis and translation about the axis perpendicular to the plane of the figure.

Family 3 – all links have the same three degrees of freedom removed. For the mechanism in Fig. 3, these are rotation about the vertical and horizontal axes and translation along an axis perpendicular to the plane of the figure. This family includes mainly planar mechanisms as well as some spatial mechanisms.

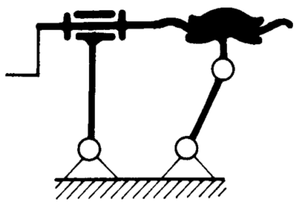

Fig. 2. Family 2 – 4 DOF in total [Miller 1989]

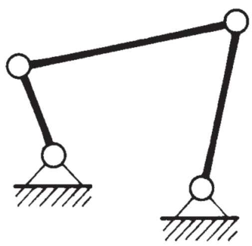

Fig. 3. Family 3 – 3 DOF in total [Miller 1989]

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Classification of mechanisms - structural

Family

Family 4 – all links retain the same two degrees of freedom. For the wedge mechanism in Fig. 1, these are displacement along the horizontal and vertical axes. This family includes sliding pair mechanisms (wedge mechanisms) and screw mechanisms.

Family 5 – link has 1 DOF (Fig. 2). Driver (or driving) links belong to this family.

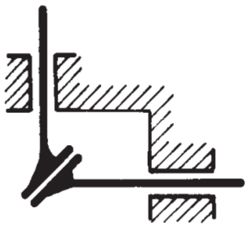

Fig. 1. Family 4 – 2 DOF in total [Miller 1989]

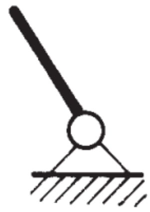

Fig. 2. Family 5 – 1 DOF in total [Miller 1989]

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Classification of mechanisms - structural

Class of mechanism (description refers to planar mechanisms)

The class of mechanism is determined by the class of structural groups (Assur groups) that comprise the mechanism. The following steps should be performed to determine the class of a mechanism:

1. Construct a kinematic diagram of the mechanism, ensuring that its mobility corresponds to the actual mobility (remove redundant degrees of freedom and links that overconstrained mechanism).
2. If class II joints are present, construct an alternative kinematic diagram in which they are replaced by class I joints.
3. Determine the class of the Assur group. The class of the mechanism is equal to the highest class of group present in the mechanism.

For some mechanisms, such as geared or complex mechanisms, it is convenient to create a structural diagram before.

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Classification of mechanisms - structural

Class of mechanism

A structural (Assur) group is a kinematic chain that jointly fulfilling the following conditions:

1. When connected to a base (fixed link) of free links, it forms a structure.
2. It is not possible to split it into simpler chains satisfy the first condition.
3. Separating a structural group from an existing mechanism does not change its mobility.

It follows from condition (1) that the Assur group has mobility equal to 0 and is described by a structural equation:

$$3n - 2p_1 = 0$$
$$3n = 2p_1$$

From this equation, the number of links and class I joints that an Assur group can have is determined (see Table).

Number of links n	2	4	6	...
Number of joints p_1	3	6	9	...

Classification of mechanisms - structural

Class of mechanism

The class of Assur group for 2 links and 3 joints is referred to as II class (one internal kinematic pair).

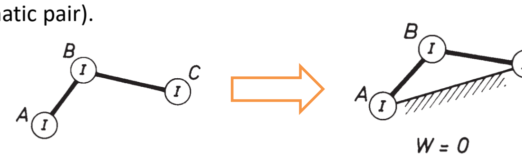


Fig. Example Assur group connected to the base [Miller 1989]

For other groups consisting of a greater numer of links, the class of the group is determined by the number of sides of the largest closed polygon within the group*.

*According to some sources [e.g. Siemieniako 1999, Miller 1989], the class of a group is determined by the number of members and kinematic pairs. The table below presents the class membership of groups according to this rule.

Class of group	*II*	*III*	*IV*	
Number of links n	2	4	6	...
Number of joints p_1	3	6	9	...

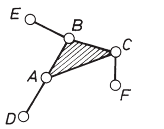


Fig. An example of a class group III [Miller 1989]

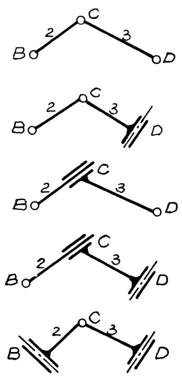
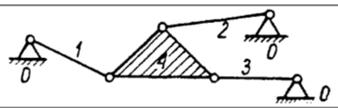
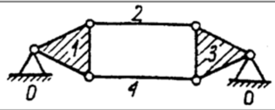
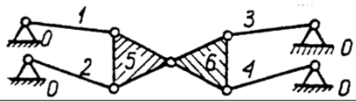
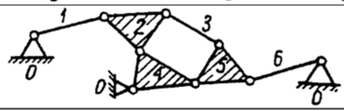


Fig. All variants of a class II group with class I joints [Młynarski 1997]

Classification of mechanisms - structural

Class of mechanism

The two rules presented for determining the class of a group, closed polygon rule the rule based on the number of links and joints, are not equivalent (the figures in the table are from [Siemieniako 1999]).

Class of group by: A closed polygon	Class of group by: Numbers of links and joints	Number of members	Number of joints	Group structural diagram
III	*III*	4	6	
IV	*III*	4	6	
III	*IV*	6	9	
IV	*IV*	6	9	

Classification of mechanisms - structural

Class of mechanism

In these classes the closed polygon rule introduced by I. I. Artobolevski [Ceccarelli 2014] will be used. Some authors refer to the drive link as a class I group.

Mechanism order

The order of a mechanism is defined as the number of exterior nodes of a group which, when connected to a base, form a structure.

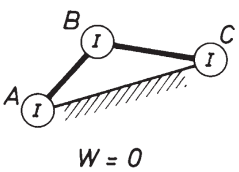


Fig. Group of order 2
[Miller 1989]

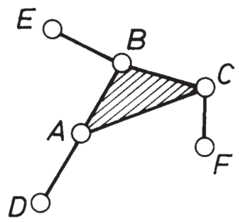


Fig. Group of order 3
[Miller 1989]

Classification of mechanisms - constructional

Constructional classification divides mechanisms according to their constructional properties.

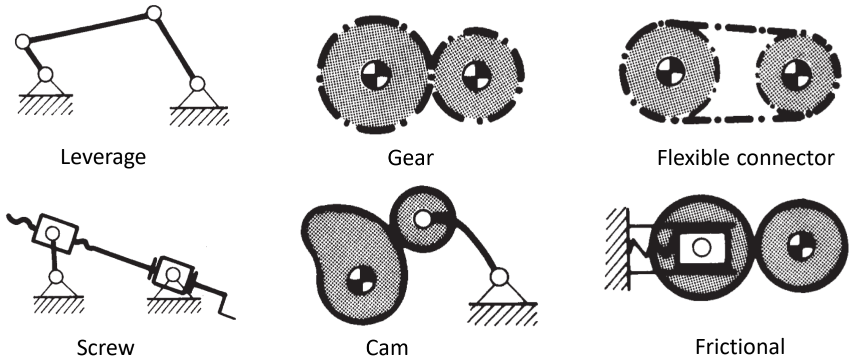


Fig. [Miller 1989]

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Classification of mechanisms - functional

Functional classification divides the mechanisms according to the tasks perform in machines. According to Artobolevski [Mlynarski, 1997], mechanisms are classified as follows: switching on and off, grippers, scales, brakes, stopping, for performing mathematical operations, steering, lever, valves, parallelograms, non-parallelograms, pistons, rotor and eccentric pumps, reels machines, regulators, with swinging discs, presses, power supply, casual power supply, heavy lifting, hammers, locking devices, timing, ratchet and other devices.

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Structural analysis of mechanisms

The structural analysis of the mechanism aims to determine its mobility, structural classification and serves as the starting point for further kinematic and dynamic analysis.

During the analysis, two types of diagrams can be made:

- *kinematic diagram* - represents the mechanism in a simplified manner, retaining the construction and characteristic geometric dimensions of the links necessary for kinematic analysis,
- *structural diagram* – represents, in a symbolic form, the connections between the links.

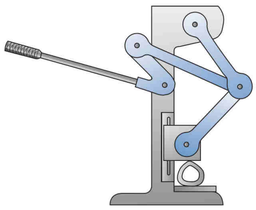


Fig. Constructional diagram of press [Myszka 2012]

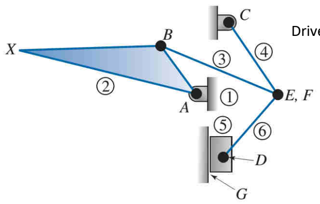


Fig. Kinematic diagram of press [Myszka 2012]

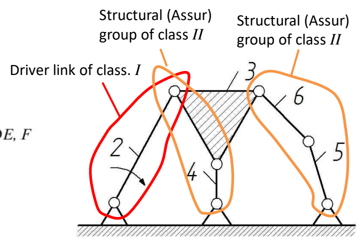


Fig. Structural diagram of press

$M = 3n - 2P_1 - P_2 = 3 \cdot 5 - 2 \cdot 7 = 15 - 14 = 1$

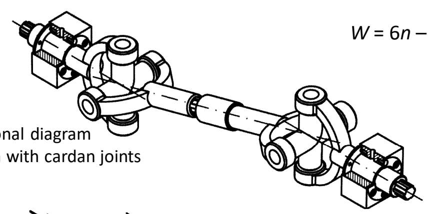
The press mechanism has a mobility of 1 and is classified as family 3, class II, order 2.

What modifications are required to reclassify it as a III class mechanism?

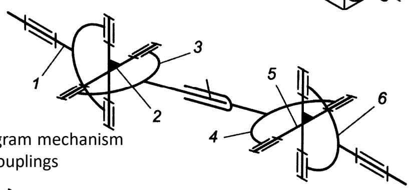
Structural analysis of mechanisms

$W = 6n - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5 = 6 \cdot 6 - 5 \cdot 7 = 36 - 35 = 1$

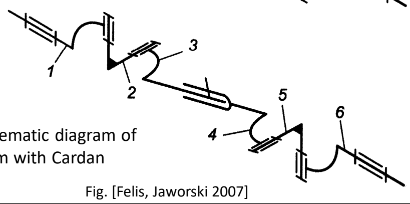
What is the class of the mechanism?



Constructional diagram mechanism with cardan joints



Kinematic diagram mechanism with Cardan couplings

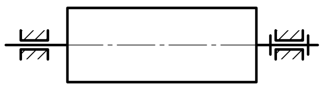


Equivalent kinematic diagram of the mechanism with Cardan couplings

Fig. [Felis, Jaworski 2007]

Linkage mechanisms

A ***Linkage mechanism**** is build exclusively from lower pairs , with the exception of screw pairs.
The simplest mechanism of this group is a rotor shaft e.g. of a fan or electric motor supported in bearings,



the basic mechanism, however, is the four bar linkage. A considerable number of linkage mechanisms can be obtained from it through inversion[1] and transformation [2].

Types of links that occur in a four bar linkage:
Crank (driver link) - can make a full rotation and is connected to the base.
Rocker (follower, lever) - performs an oscillating-rotating (pendulum) motion and is connected to the base.
Coupler (connecting arm) – undergoes plane motion and is not connected to the base.
Base (frame, fixed link) - remains stationary relative to the chosen reference system.

[1]*Inversion* (*kinematic inversion*) – the change of the base from one link to another e.g. crank with base
[2]*Transformation* – the change of the type of joint e.g. from a revolute to a sliding joint.

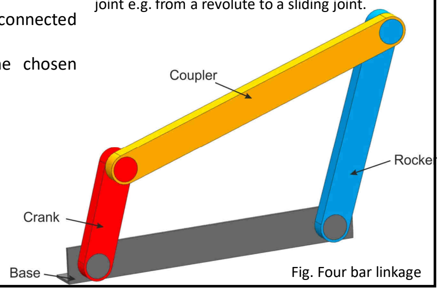


Fig. Four bar linkage

*There is no single definition of linkage mechanism. Presented definition is not universally accepted and narrows scope of mechanism that can be assigned to this group.
Linkage – *mechanism having only lower pairs* [Uicker 2011]
Linkage – *mechanisms consisting of lower pairs only* or *loosely synonymous to the term mechanism* [Ambekar 2007]
Linkage – *is a mechanism where rigid parts are connected together to form a chain* [Myszka 2012]

Linkage mechanisms

From a practical point of view, it is important to know the conditions that must be satisfied by the links of the four bar linkage to allow one of them to perform a full rotation. In such case, a motor can drive this link. The answer to this question is given by Grashof's law, which states that for a four-bar linkage, if the sum of the shortest and longest links is not greater than the sum of the remaining two links, at least one of the links will be capable of revolving. Grashof's law (condition) is exp

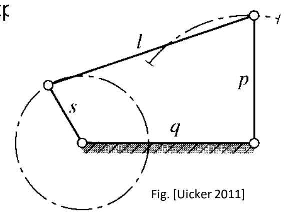


Fig. [Uicker 2011]

$s + l \leq p + q$

s = length of shortest link
l = length of longest link
p = length of one remaining link
q = length of other remaining link

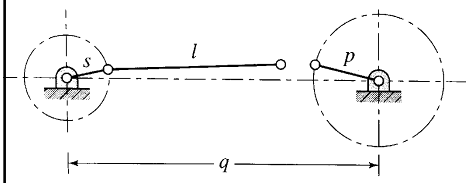
Reuleaux approached the problem somewhat differently but, naturally, obtains the same results. The conditions are as follows:
$s + l + p \geq q$
$s + l - p \leq q$
$s + q + p \geq l$
$s + q - p \leq l$

The result is of course the same.

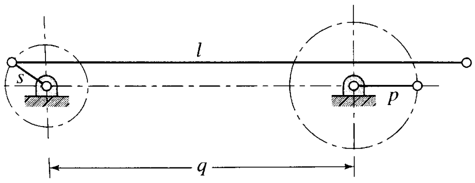
Linkage mechanisms

These four condition are illustrated below showing what occurs if they are not met.

Implementation is not feasible

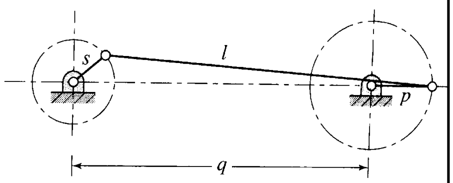


$s + l + p < q$

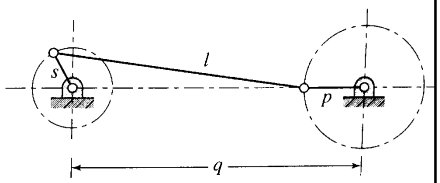


$s + q + p < l$

Link *s* is not a crank



$s + l - p > q$



$s + q - p < l$

Fig. [Uicker 2011]

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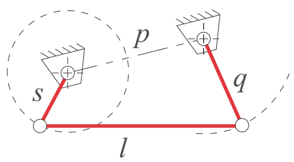
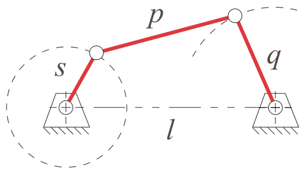
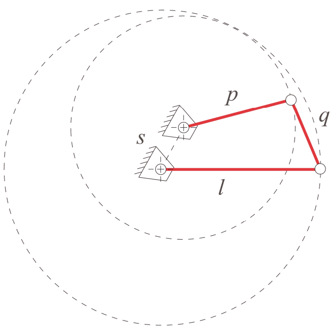
Linkage mechanisms

Depending on the functions performed by the links (through inverstion), three types of mechanisms are obtained.





Crank-rocker



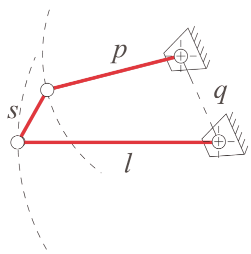


Fig. [Norton 1999]

Double crank

Double rocker

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Linkage mechanisms

By transforming the revloute joint to a sliding joint in the four bar mechanism, a slider-crank mechanism is obtained (Fig. 1). It is commonly used in internal combustion engines and compressors.

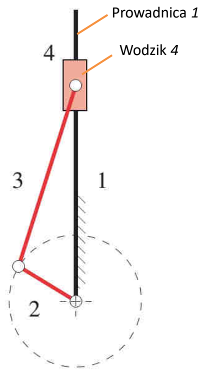

Fig. 1. Slider-crank linkage [Norton 1999]

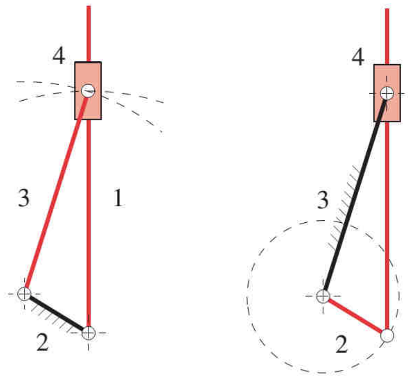

Fig. 2. Inversion of the slider-crank linkage [Norton 1999]

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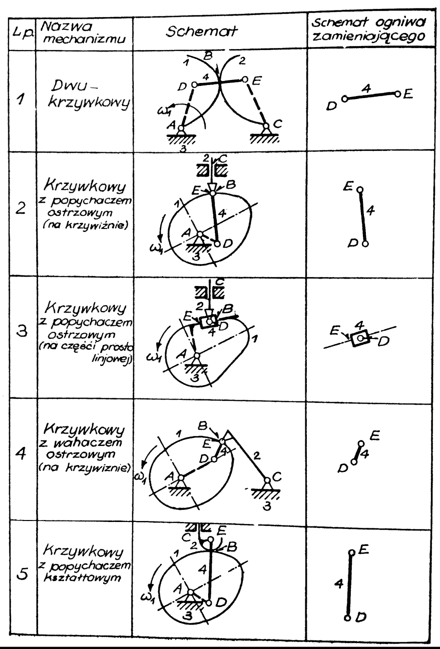
Linkage mechanisms

Converting kinematic pairs of class II to class I pairs allows a kinematic analysis to be performed using the methods applied to linkage mechanisms. The resulting equivalent mechanism must be kinematically equivalent to the real mechanism at the given position. An additional link is introduced so that the mobility of the mechanism remains unchanged. In the case of cam mechanisms, the ends of the link are at the centers of the adjoining curves.



Lp.	Nazwa mechanizmu	Schemat	Schemat ogniwa zamieniającego
1	Dwu-krzywkowy		
2	Krzywkowy z popychaczem ostrzowym (na krzywiźnie)		
3	Krzywkowy z popychaczem ostrzowym (na części prostoliniowej)		
4	Krzywkowy z wahaczem ostrzowym (na krzywiźnie)		
5	Krzywkowy z popychaczem kształtowym		

Fig. [Młynarski 1997]

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Linkage mechanisms

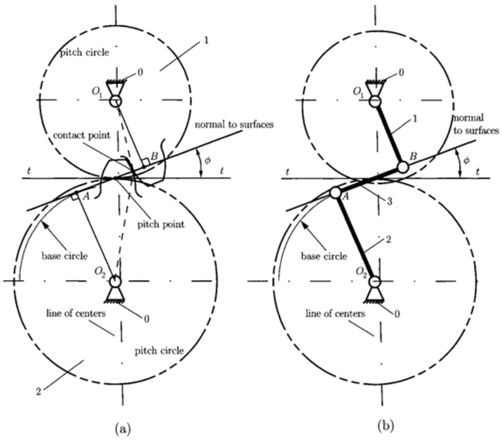

Fig. [Marghitu 2005]

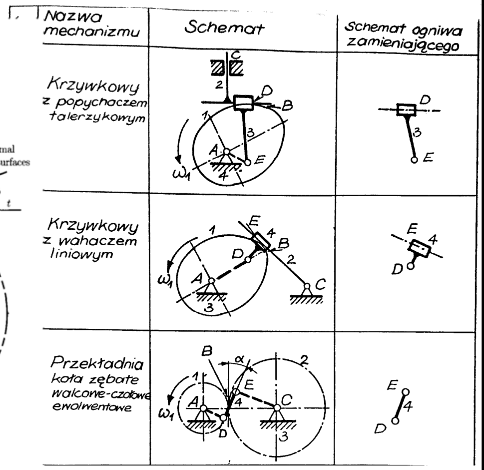

Fig. [Młynarski 1997]

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Linkage mechanisms

Examples

Four bar linkage (parallelogram) used in the transmission of wheel drive

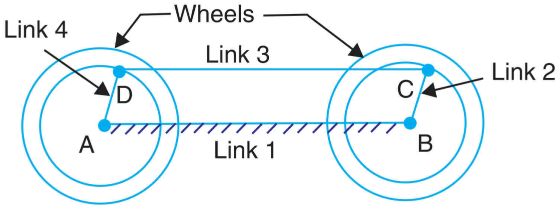

Fig. [Khurmi 2005]

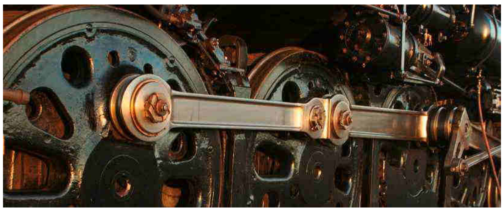

Fig. [https://en.wikipedia.org/wiki/Coupling_rod]

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Linkage mechanisms

Examples

Beam engine

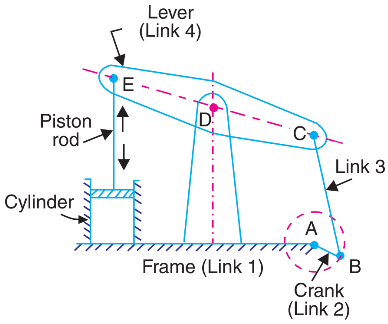

Fig. [Khurmi 2005]

Fig. [https://hanselygretel.eu/tienda/maquina-vapor-watt/]

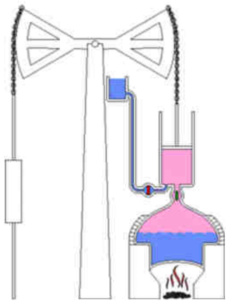

Fig. [https://en.wikipedia.org/wiki/Thomas_Newcomen]

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Linkage mechanisms

Examples

Slidr crank mechanism

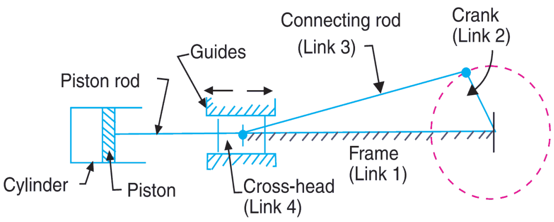

Fig. [Khurmi 2005]

Fig. [http://www.magoda.com/industrial/oem-manufacturing-in-the-united-states/]

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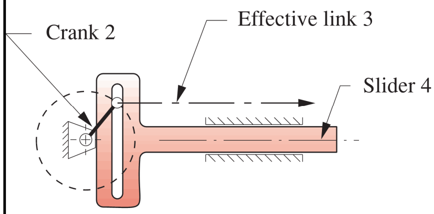
Linkage mechanisms

Examples

Scotch yoke mechanisn



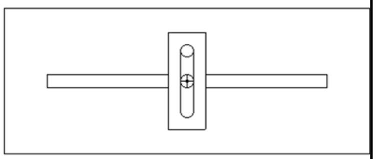


Fig. [https://en.wikipedia.org/wiki/Scotch_yoke#
/media/File:Scotch_yoke_animation.gif]

Fig. [Norton 1999]

Fig. Scotch yoke mechanism [http://www.etotheipiplusone.net/?m=201209]

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Examples

Geneva mechanism

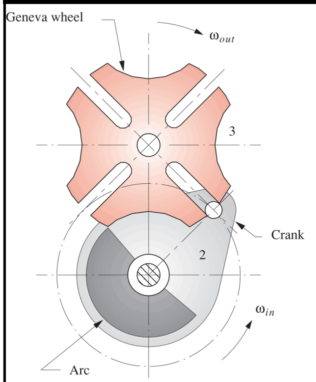

Fig. Geneva mechanism [Norton 1999]

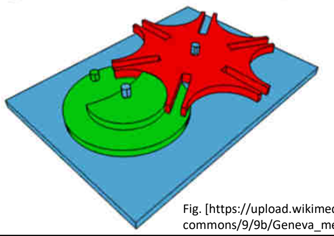

Fig. [https://upload.wikimedia.org/wikipedia/
commons/9/9b/Geneva_mechanism_6spoke_animation.gif]

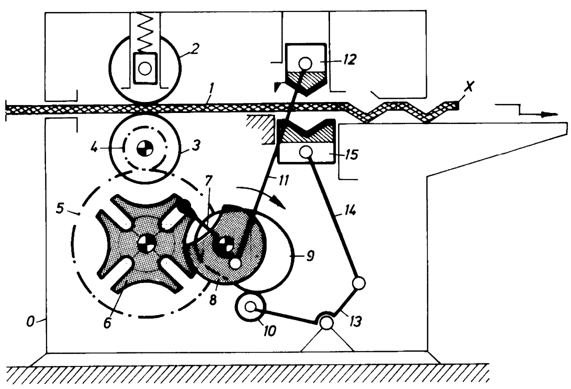

Fig. Sheet metal processing machine
[Miller 1989]

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Linkage mechanisms

Examples

Straight-line mechanism

In the 17th century, before the development of milling machines, it was difficult to achieve precise sliding pairs. Movement along a straight line was accomplished through the use of relvolute pairs. Straight-lines mechanism are part of steering mechanisms, whose purpose is to move along a predetermined trajectory.

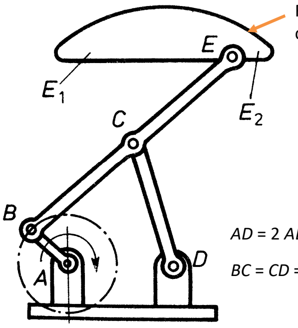

Fig. Chebyschev-Hoekens linkage (approximate straight line) [Morecki 1987]

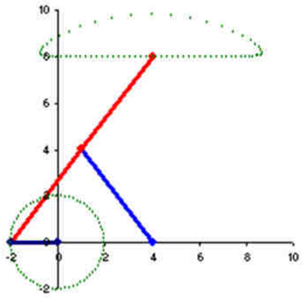

Fig. [https://upload.wikimedia.org/wikipedia /commons/b/be/Hoekens_linkage_animated.gi]

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Linkage mechanisms

Examples

Straight-line mechanism

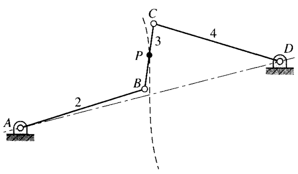

Fig. Wattsa linkage (approximate straight line) [Uicker 2011]

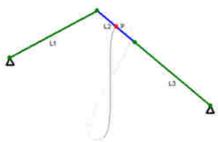

Fig. [https://upload.wikimedia.org/wikipedia/commons/9/93/Watts_linkage.gif]

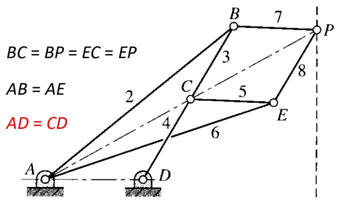

Fig. Peaucellier-Lipkin linkage (exact straight line) [Uicker 2011]

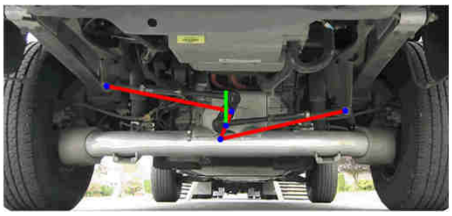

Fig. [https://en.wikipedia.org/wiki/Watt%27s_linkage# /media/File:Watt%27s_Linkage_Rear_Suspension.gif]

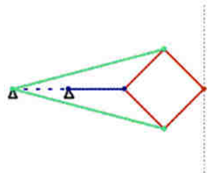

Fig. [https://en.wikipedia.org/wiki/Straight_line_ mechanism#/media/File:Peaucellier_linkage_animation.gif]

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Linkage mechanisms

Examples

Straight-line mechanism

AD = 4, AB = CD = 5, BC = 2

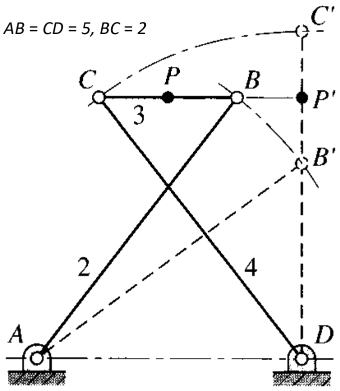

Fig. Chebyschev linkage (approximate straight line) [Uicker 2011]

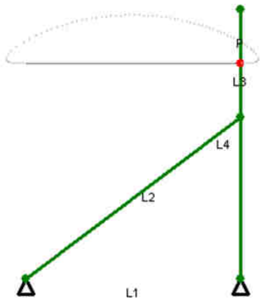

Fig. [https://en.wikipedia.org/wiki/Chebyshev_linkage#/media/File:Chebyshev_linkage.g]if

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Linkage mechanisms

Examples

Straight-line mechanism

BC = AD/2

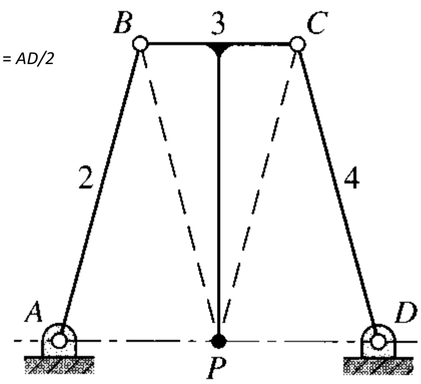

Fig. Roberts linkage (approximate straight line) [Uicker 2011]

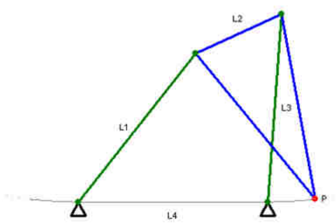

Fig. [https://en.wikipedia.org/wiki/Roberts_Mechanism#/media/File:Roberts_linkage.gif]

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Linkage mechanisms

Examples

Straight-line mechanism

Fig. [https://pt.depositphotos.com/2975201
/stock-photo-the-port-crane.html]

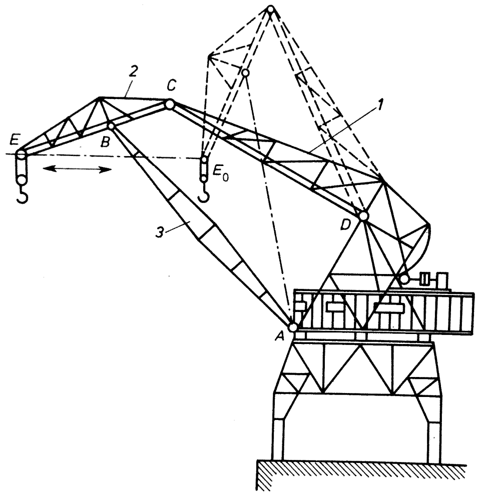

Fig. Port crane, coupler curve EE_0
approximate straight line [Morecki 1987]

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Linkage mechanisms

Examples

Straight-line mechanism

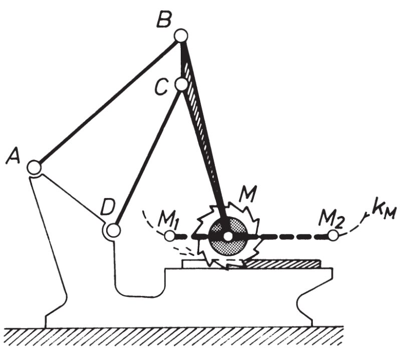

Fig. coupler curve M_1M_2 approximate straight line
[Miller 1989]

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Linkage mechanisms

Examples

Cardana (Hook, universal) joint (variable velocity joint)

Spherical four bar linkage

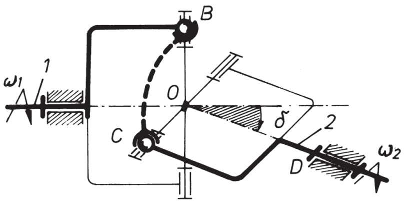

Fig. [Miller 1989]

Fig. Double Cardan joint [agromer.pl/ogolna-wal-kardana-john-deere-al117415,c2,p5717,pl.html]

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Linkage mechanisms

Examples

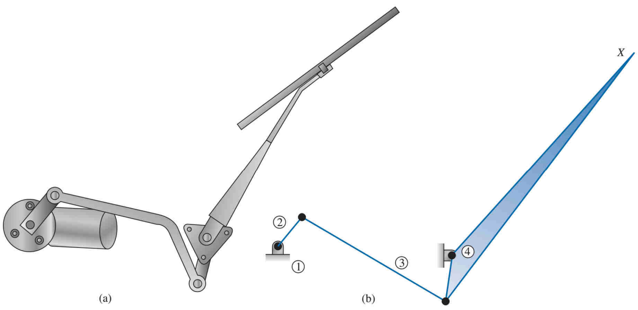

FIGURE 1.33 Rear-window wiper mechanism.

Fig. [Myszka 2012]

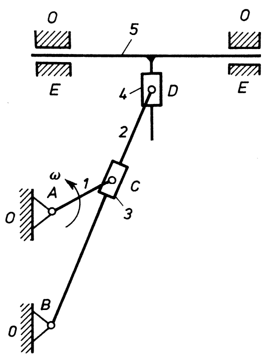

Fig. The drive mechanism of the cross planer table [Morecki 1987]

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