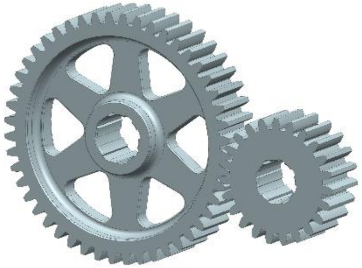

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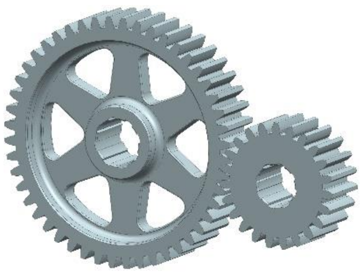
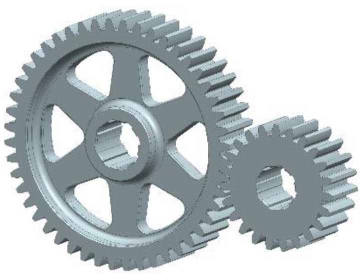
Machine Parts II

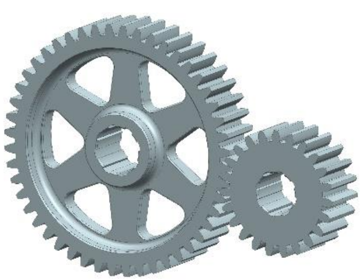


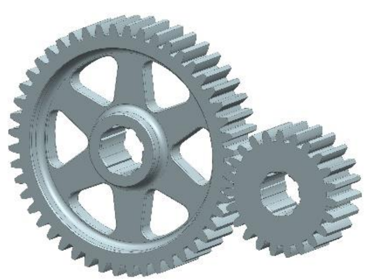
Spur and helical gears

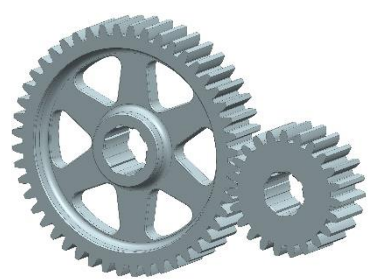
Beta version

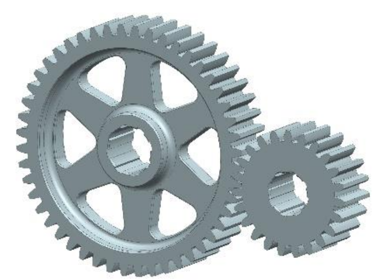
Basic properties, terminology, dimensions, forces and manufacturing











Lublin 2021

Łukasz Jedliński, Ph.D., Eng.

Introduction

Gear units are used to adjust torque and rotational speed generated by source to be suitable for output device. Additionally, they are used to change the direction of transmitting torque.

Main components of simple gear unit: gears, bearings, shafts, housing, oil and seals.

Fig. [https://www.shreecongear.com/gearbox-3/]

Introduction

Gear units can work as **reducers** (speed of input shaft is greater than output shaft and torque on output shaft is greater than torque on input shaft) or **multipliers** (speed of input shaft is smaller than output shaft and torque on output shaft is smaller than torque on input shaft)

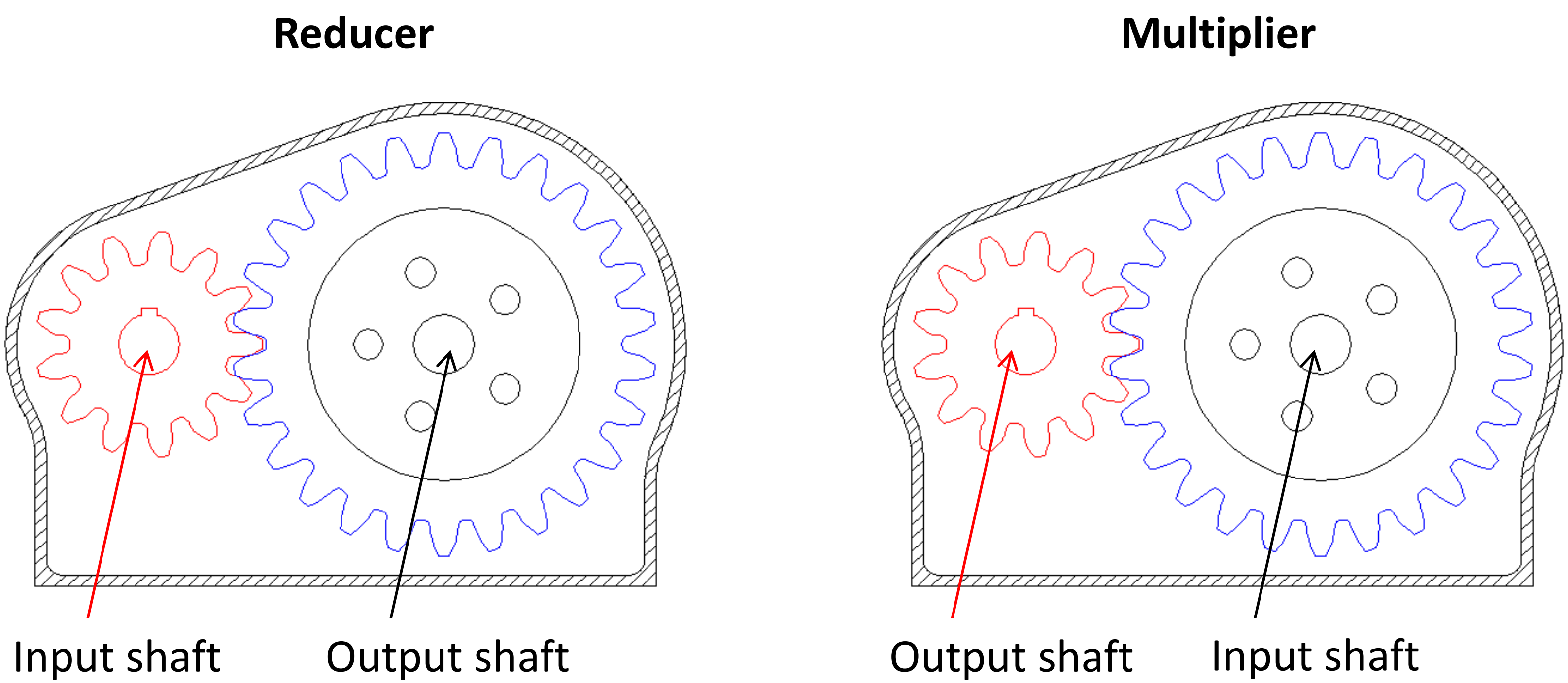


Fig. [https://en.wikipedia.org/wiki/Transmission_(mechanics)#/media/File:Gear_reducer.gif]

Introduction

I. Classification of gears according to the position of gears axis of rotation and line of tooth*:

1. ***Parallel axes***:
   - spur gears,
   - helical gears,
   - double helical gears,
   - rack nad pinion,
   - strain wave gearing,
   - planetary (epicyclic) gear unit with cylindrical gears.

2. ***Intersecting axes***:
   - straight bevel gears,
   - spiral bevel gears,
   - face gears,
   - herringbone bevel gears (historically solution),
   - planetary (epicyclic) gear unit with bevel gears,

3. ***Twist axes*** (axes are not laying in one plane):
   - hypoid gears,
   - spiroid, planoid, helicon gears,
   - worm drive,
   - crossed helical (screw) gears.

Fig. Some types of gears [Mott 2018]

**Line of tooth* – line or curve created by intersection of pitch cylinder or cone and tooth working surface

Introduction

II. Classification of gears according to the possibility of movement of gears axis of rotation regarding to housing and line of tooth:

1. Fixed axes:

- spur gears,
- helical gears,
- double helical gears,
- rack and pinion,
- strain wave gearing,
- straight bevel gears,
- spiral bevel gears,
- face gears,
- herringbone bevel gears (historically solution),
- hypoid gears,
- spiroid, planoid, helicon gears,
- worm drive,
- crossed helical (screw) gears.

2. Movable axis:

- Planetary (epicyclic) gear unit.

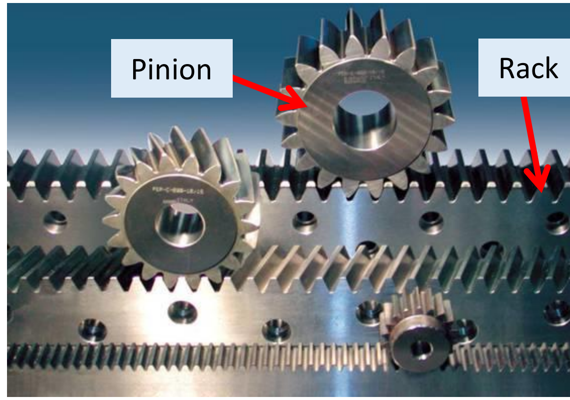


Fig. Rack and pinion [http://www.directindustry.com/industrial-manufacturer/helical-toothed-rack-pinion-215774.html]

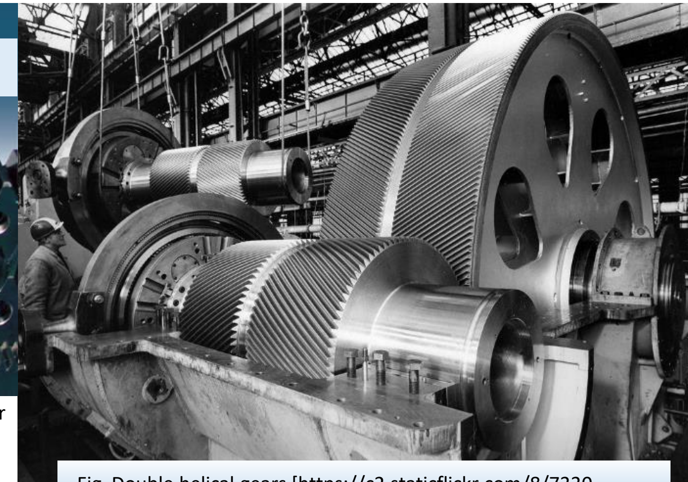


Fig. Double helical gears [https://c2.staticflickr.com/8/7330/14068135536_8b2edbf5e3_b.jpg]

Fig. Herringbone bevel gears (made by Citroen) [https://en.wikipedia.org/wiki/Herringbone_gear]

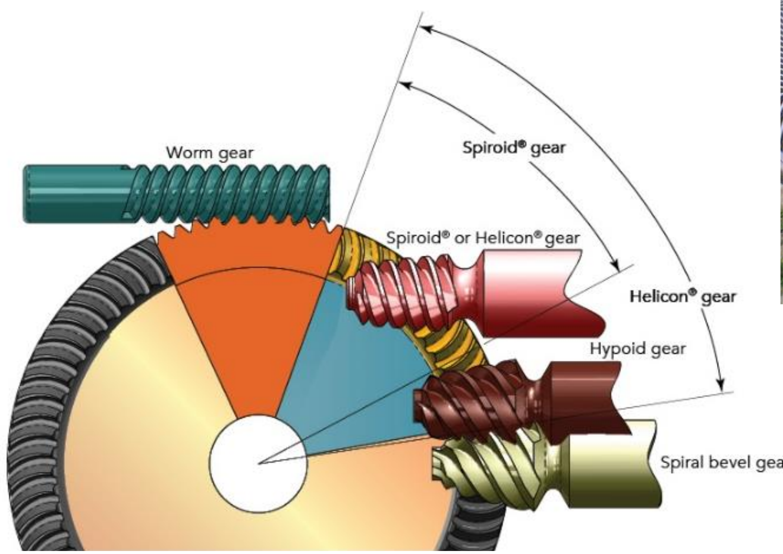


Fig. [https://www.motioncontroltips.com/what-is-a-gear/]

II. Classification of gears according to the possibility of movement of gears axis of rotation regarding to housing and line of tooth:

1. Fixed axes:
- spur gears,
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- double helical gears,
- rack and pinion,
- strain wave gearing,
- straight bevel gears,
- spiral bevel gears,
- face gears,
- herringbone bevel gears (historically solution),
- hypoid gears,
- spiroid, planoid, helicon gears,
- worm drive.

2. Movable axis:
- Planetary (epicyclic) gear unit.

Fig. Strain wave gearing [https://www.gamweb.com/strain-wave-operation.html]

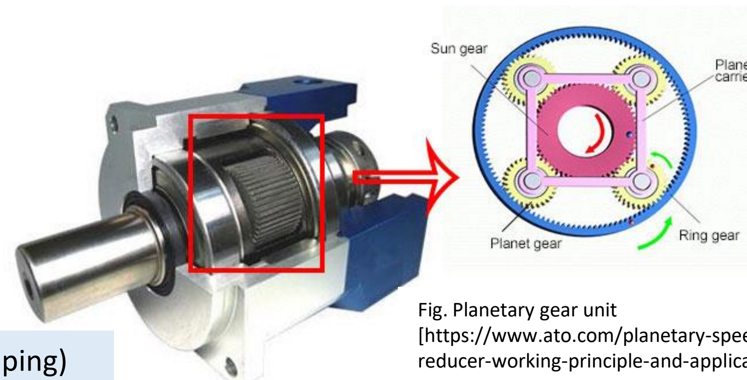


Fig. Planetary gear unit [https://www.ato.com/planetary-speed-reducer-working-principle-and-applications /]

Fig. Face gears [https://www.indiamart.com/proddetail/face-gears-8082228433.html]

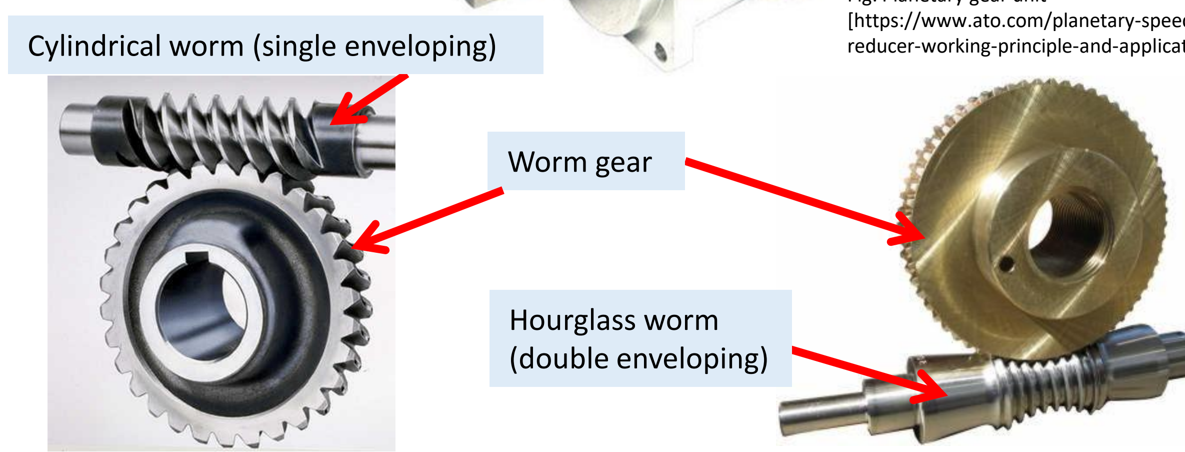


Fig. [https://www.indiamart.com/gp-engineering-fabrics/]

Fig. [http://www.globalcncindia.in/industrial-gears.html]

Introduction

Noncircular gears

Gears can also be used to obtain nonlinear dependence (displacement, velocity) between input gear and output gear

Fig. [Radzevich 2018]

Fig. [Radzevich 2018]

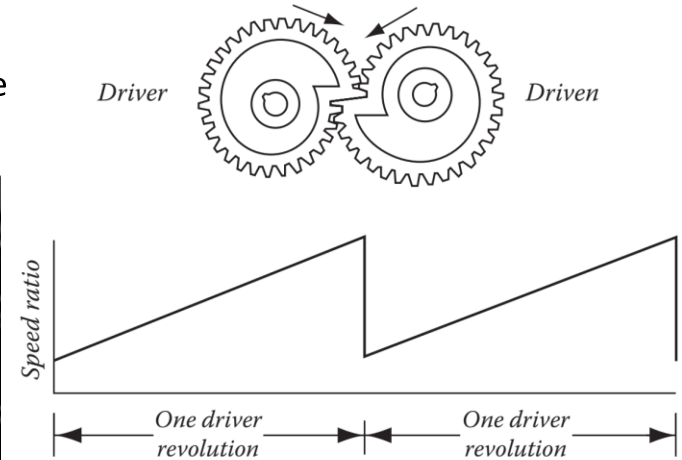
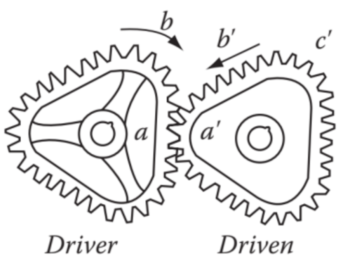


Fig. Scroll gears and kinemtaic characteristic [Radzevich 2012]



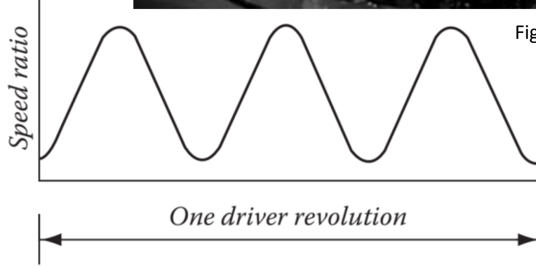
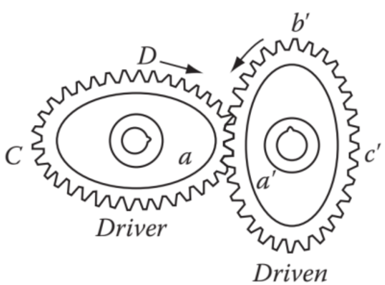


Fig. Triangular gears and kinematic characteristic [Radzevich 2012]



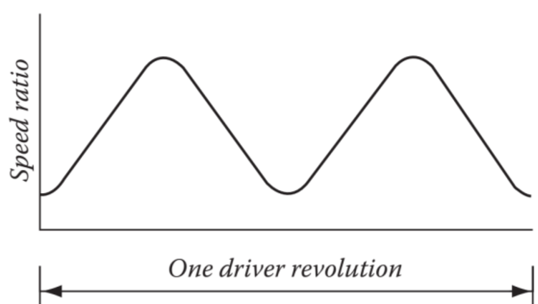
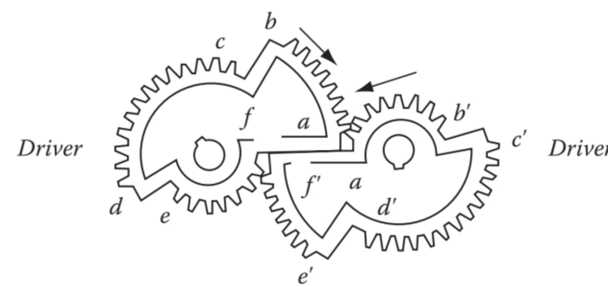


Fig. Elliptical gears and kinematic characteristic [Radzevich 2012]



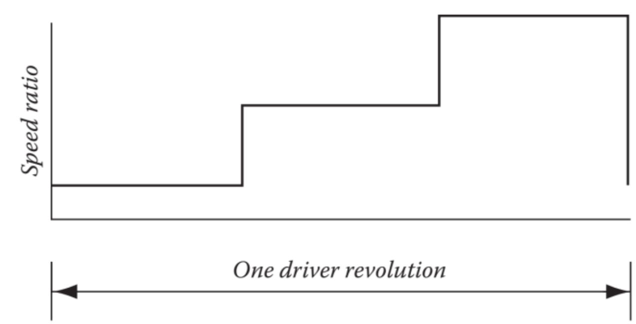


Fig. Multiple-sector gears and kinemtaic characteristic [Radzevich 2012]

Introduction

Main properties of gear types [Radzevich 2012]

Type of Gear	Approx. Range of Efficiency	Max[a] Width Generally Used[b]	Type of Load Imposed on Support Bearings	Nominal Range of Reduction Ratio[c]	Nominal Max Pitch Line Velocity, fpm (5.08 × 10^{-3} m/s)		Member of Pair	Methods of Manufacture	Methods of Refining
					Aircraft and High Precision	Commercial			
Parallel Axes									
External spur gears	97–99.5	$F = d$	Radial	1:1–5:1	20,000	4,000	Both	Hob, shape, mill, broach, stamp, sinter	Grind, shave, lap (crossed axis only), hone
Internal spur gears	97–99.5	Directed by mating gear	Radial	1:5–7:1	20,000	4,000		Shape, mill, broach, stamp, sinter	Grind, shave, lap (crossed axis only), hone
External helical gears	97–99.5	$F = d$	Radial and thrust	1:1–10:1	40,000	4,000	Both	Hob, shape, mill	Grind, shave, lap, hone
Internal helical gears	97–99.5	Directed by mating gear	Radial and thrust	5:1–10:1	20,000	4,000		Shape, mill	Shave, grind, lap, hone
Internal herringbone or double helical	97–99.5	Directed by mating gear	Radial	2:1–20:1	20,000	4,000		Shape	Lap, shave, hone
External herringbone or double helical	97–99.5	$F = 2d$	Radial	1:1–20:1	40,000	4,000	Both	Hob, shape, mill	Grind, shave, lap, hone
Intersecting Axis									
Straight bevel gear	97–99.5	1/3 cone distance	Radial and thrust	1:1–8:1	10,000	1,000	Both	Generating forming	Grind
Zerol bevel gear	97–99.5	28% of cone distance	Radial and thrust	1:1–8:1	10,000	1,000	Both Gear	Generating forming	Grind

Introduction

Main properties of gear types [Radzevich 2012]

Spiral bevel gear	97–99.5	1/3 cone	Radial and thrust	1:1–8:1	25,000	4,000	Both Gear	Generating forming	Grind, lap
Face gear	95–99.5	From 0.2d at a low ratio to d at a high ratio	Radial and thrust	3:1–8:1	5,000	4,000	Pinion	Same as external spur gear	Lap, grind, shave, hone
							Gear	Shape	Shape, lap
Beveloid	95–99.5	$5/p_n$	Radial and thrust	1:1–8:1	5,000	4,000	Both	Hob generating	Lap, grind
Crossed-helical	50–95	$F_1 = 4p_n \sin \psi_1$ $F_2 = 4p_n \sin \psi_2$	Radial and thrust	1:1–100:1	10,000	4,000	Both	Hob, shape, mill	Grind, lap, shave
Cylindrical worm	50–90	$F_w = 5p_n \cos \lambda$ $F_g = 0.67d$	Radial and thrust	3:1–100:1	10,000	5,000	Pinion	Mill, hob	Grind
							Gear	Hob	Lap
Double-enveloping worm	50–98	$F_w = 0.9D$ $F_g = 0.9d$	Radial and thrust	3:1–100:1	10,000	4,000	Worm	Shape, hob	Lap, grind
							Wheel	Hob, mill	
Hypoid	90–98	$F_g = 1/3$ cone distance	Radial and thrust	1:1–10:1	10,000	4,000	Both	Generating Forming	Grind, lap
High-reduction hypoid	50–90	$F_g = 0.15D$	Radial and thrust	10:1–50:1	10,000	4,000	Pinion	Generating	Grind, lap
							Gear	Forming	Grind, lap
Spiroid	50–97	$F_p = 0.24D$ $F_g = 0.14D$	Radial and thrust	9:1–100:1	10,000	6,000	Pinion	Mill, hob	Grind, chase
							Gear	Hob, mold	
Planoid	90–98	$F = 1/3$ cone distance	Radial and thrust	1.5:1–10:1	10,000	4,000	Pinion	Hob	Lap, grind
							Gear	Mill, broach	Grind
Helicon	50–98	$F_p = 0.21D$ $F_g = 0.12D$	Radial and thrust	3:1–100:1	10,000	6,000	Pinion	Mill, hob	Grind, chase
							Gear	Hob, mold	
Face gear	95–99.5	From 0.2D at low ratio to D at a high ratio	Radial and thrust	3:1–8:1	10,000	4,000	Pinion	Same as external spur	Same as external spur
							Gear	Shape	Lap
Beveloid	50–95	$5/p_n$	Radial and thrust	1:1–100:1	10,000	4,000	Both	Generating, hob	Grind, lap

[a] Face width given is only a "nominal" maximum. Consult other parts of the *Gear Handbook* for detail limitations on face width.
[b] d = pinion pitch diameter; D = gear pitch diameter; λ = lead angle; ψ = helix angle; p_n = normal circular pitch.
[c] Gear types showing an upper ratio of 100:1 can be built to 300:1 or higher. The ratio of 10:1 is shown as a normal maximum limit.

Gear terminology – involute profile

Teeth profile determines very important properties like: strength and durability (contact stress, bending stress, slipping velocity, accuracy of manufacture→ possible methods of manufacturing, tolerance for manufacture and assembly errors), cost (possible methods of manufacturing), efficency (slipping velocity), noise and vibration (accuracy of manufacture, tolerance for manufacture and assembly errors) etc.

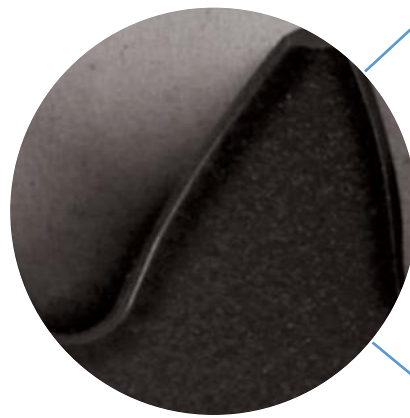


Fig. Picture in the circle [Radzevich 2016]

Involute profile
- Most common

Cycloidal profile
- Older than involute

Novikov–Wildhaber gears
- Newer than involute

Gear terminology – involute profile

Cycloidal profile

Cycloidal profile is the predecessor of involute profile. It is still used to carry a motion not a power, so it is used in mechanical clocks/watches, pumps and blowers. Main disadvantages of cycloidal profile is requirement of assurance of nominal centre distance, normal force have variable direction (noise, vibration and additional dynamic load) and are difficult to manufacture. The advantage of this profle is cooperation of convex and concave parts of profiles.

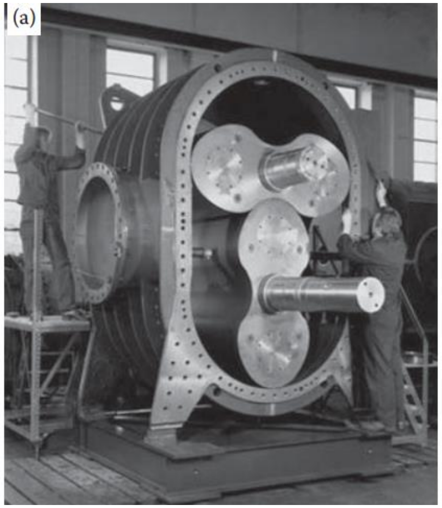


Fig. Roots blower (cycloidal profile) [Radzevich 2018]

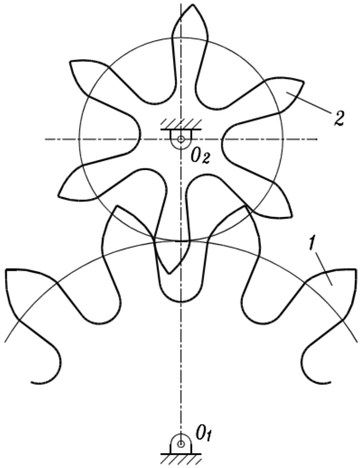


Fig. Watch gears (cycloidal profile) [Litvin 2004]

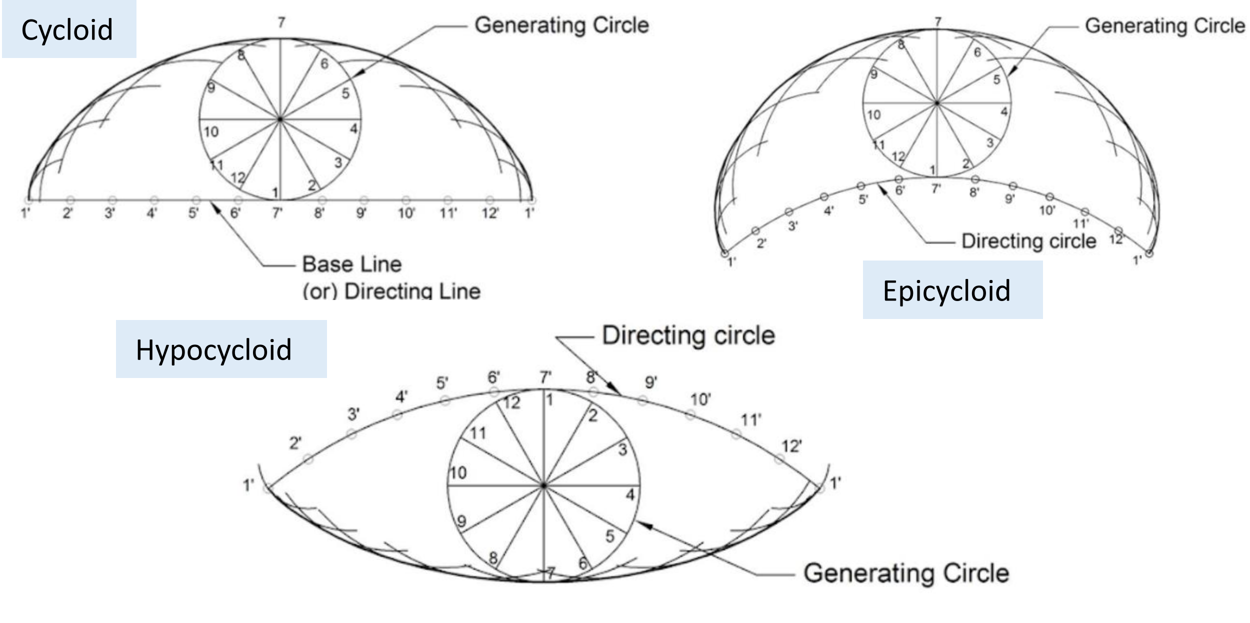


Fig. Cycloid curves [http://machine-drawing.blogspot.com/2021/01/cycloid-epicycloid-hypocycloid-arc-of.html]

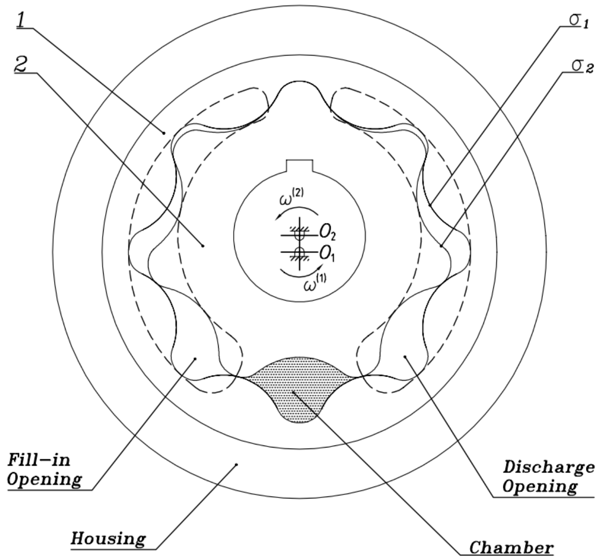


Fig. Pump (cycloidal profile) [Litvin 2004]

Gear terminology – involute profile

Novikov-Wildhaber gears

The most modern and knowable is Novikov-Wildhaber gears (circular arc profile) used in helical gears. It was developed to reduce contact stress and carry high power - convex profile is in contact with concave. Novikov-Wildhaber gears are still devloped. They are not used world wide but for example in Russia are produced gear units with this type of gears. The main disadvantages are: sensitivity to varying center distance, more complex manufacturing and high value of axial force. Advantages: high load capacity, high ratio and efficiency.

Ernst Wildhaber (Switzerland) – first elaboration, patent in 1926
Mikhail L. Novikov (Russia) modified (developed?) circular arc profile what can be practically applied – patent in 1956

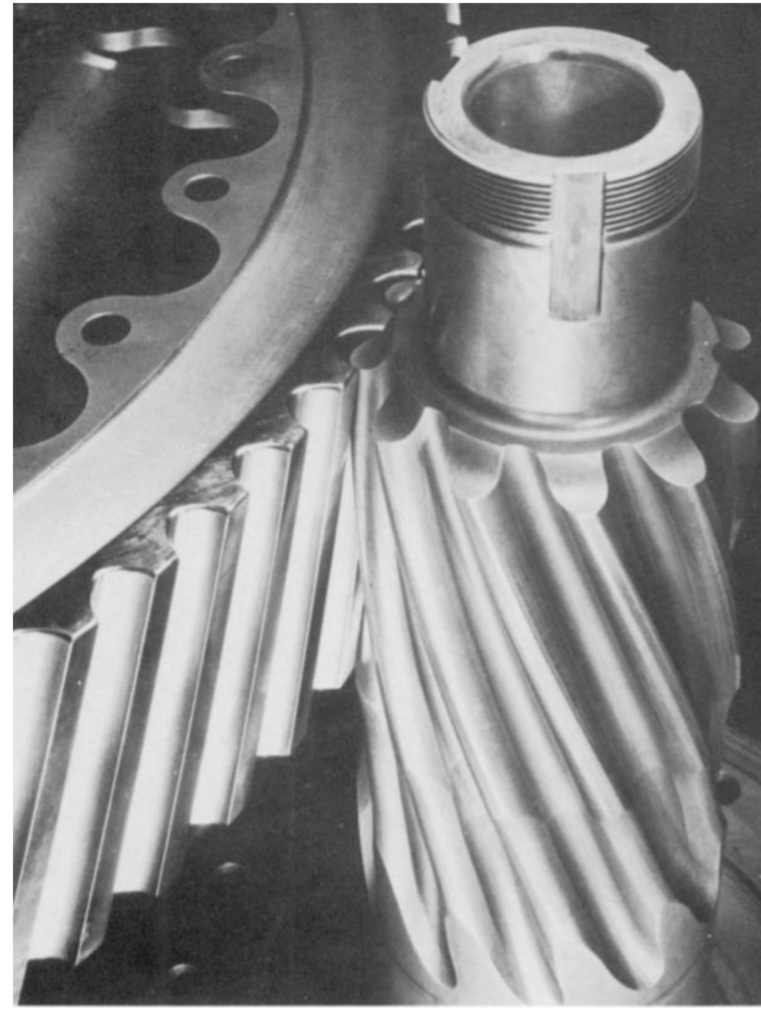


Fig. Novikov-Wildhaber gears in helicopter application [Dyson 1986: https://www.jstor.org/stable/pdf/2397915.pdf]

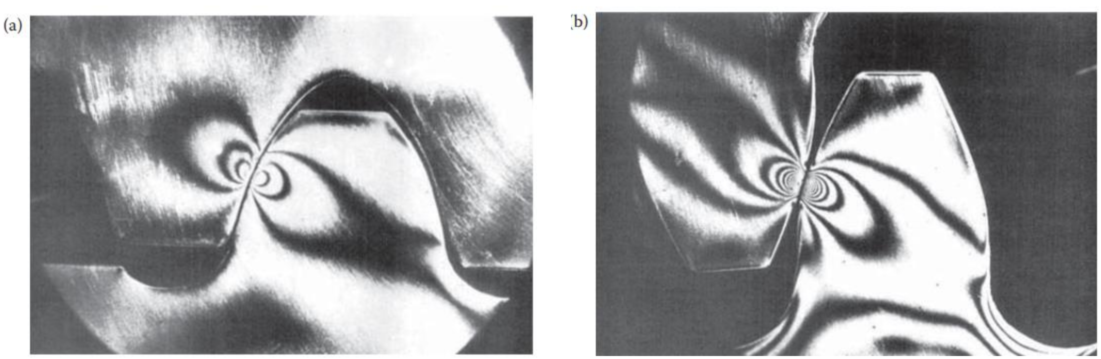


Fig.s. a) Novikov gearing, b) involute profiles gears [Radzevich 2018]

Involute profile

Currently in cylindrical gears is mainly used involute profile. It has following advantages: constant direction of normal force along line of action, constant ratio, high efficiency, variation in center distance do not have influence on working conditions form kinematic point of view, easy to manufacture, susceptible to quality control. Dissadvantages: high contact stress (convex surfaces, small area of contact), quite high slipping velocity.
Involute profile is modified to improve its properties.

Leonhard Euler (1707 - 1783), Swiss mathematician – invloute profile in gears

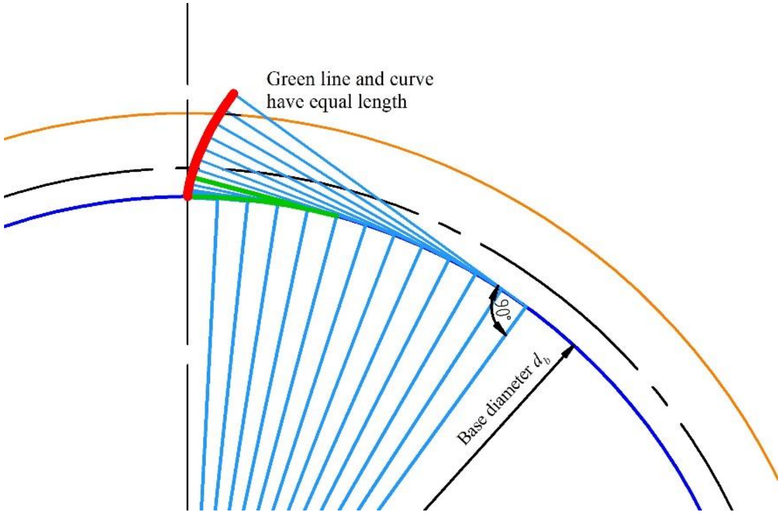


Fig. Geometrical construction of involute curve

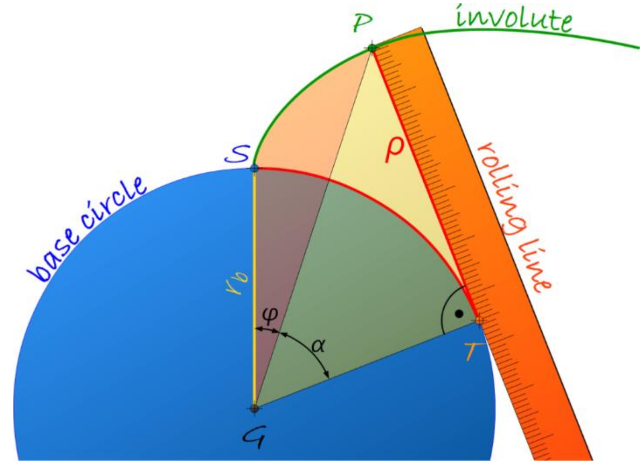


Fig. Idea of creating of involute curve
[https://www.tec-science.com/mechanical-power-transmission/involute-gear/calculation-of-involute-gears/]

Gear terminology – involute profile

Involute profile

$M_oP = MP$ – it results from definition of involute

$$M_oP = r_b(\hat{\gamma} + \hat{\alpha})$$
$$MP = r_b \tan\alpha$$

$\hat{\alpha}$ In radians

$$r_b(\hat{\gamma} + \hat{\alpha}) = r_b \tan\alpha$$

$$\hat{\gamma} = \tan\alpha - \hat{\alpha}$$

$$\boxed{inv\alpha = \tan\alpha - \hat{\alpha}}$$

involute function

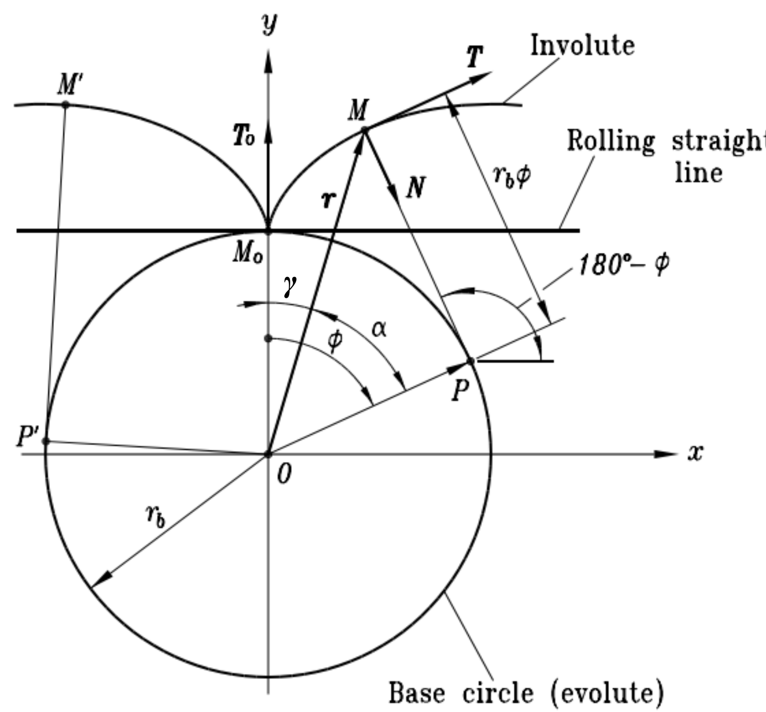


Fig. Determination of involute function [Litvin 2004]

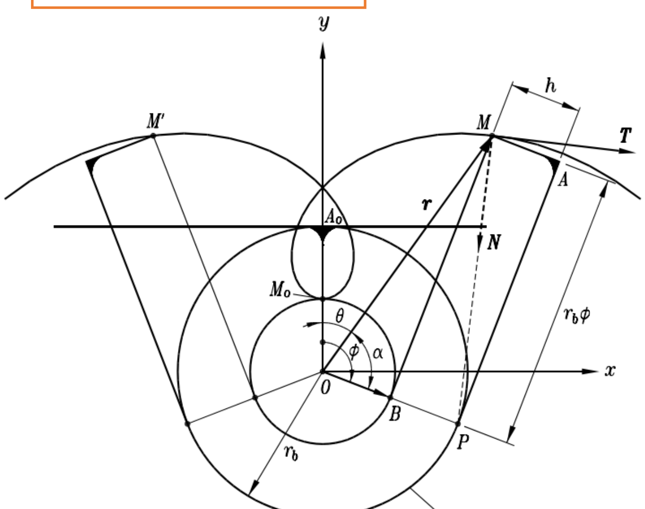


Fig. Extended involute curve [Litvin 2004]

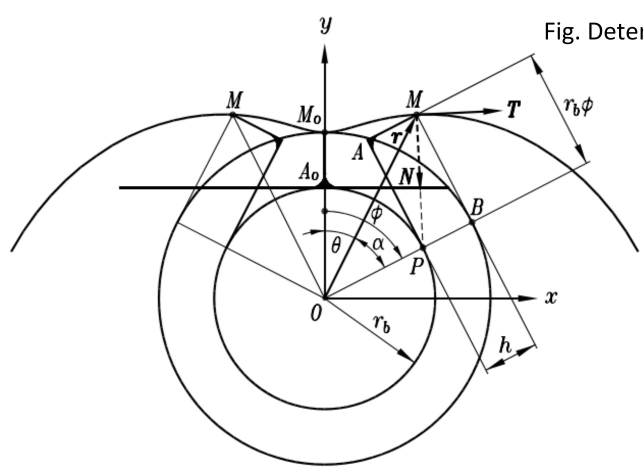


Fig. Shortened involute curve [Litvin 2004]

In cartesian coordinate system involute profile has coordinates:

$$x = r_b(\sin\varphi - \hat{\varphi}\cos\varphi)$$
$$y = r_b(\cos\varphi + \hat{\varphi}\sin\varphi)$$

Spur gear terminology (involute profile)

Spur gears are the most simple to manufacture and all methods, that are used to produce gears in general, can be applied. As a result they are also very economical. The efficiency is one the highest and they are generate only radial load to bearings. Construction of gear unit can be simple and compact. The main disadvantages are noise and vibration generated during work. This limiting the use of this type of gears to slow speed application if noise and vibration are the main concern.

Pinion is defined as a smaller (has smaller numer of teeth) of the two mating gears and ***gear*** is the bigger one.
Two mating gears are named as *pair of gears*.

Fig. [Mott 2018]

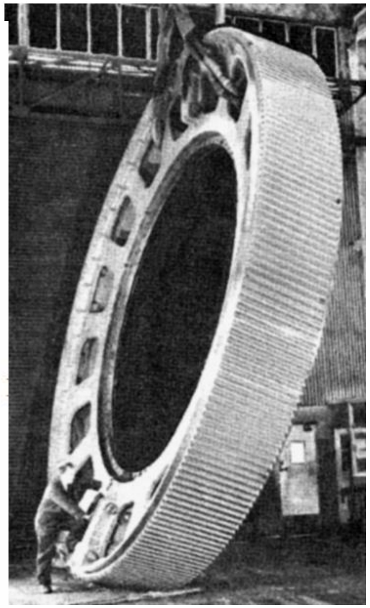


Fig. Gear for a large mill application [Radzevich 2012]

Spur gear terminology (involute profile)

Module *m* [mm] – it is one of the main parameters that characterize dimensions of gear and teeth

Pitch diameter *d* [mm] – it is an abstract diameter that is used to determine teeth dimensions.
On this diameter circular tooth thickness and tooth space are equal (for gears withouth profile shift). In practical application backlash is needed to correct work of teeth hence tooth space is slightly greater.
Pitch diameter devides height of tooth on addendum and dedendum.

Using formulas on circumference of circle equation is obatined:

$$\pi d = \underbrace{z}_{\text{Number of teeth}}\,\underbrace{p_c}_{\text{Circular pitch}} \rightarrow d = \underbrace{\boxed{\frac{p_c}{\pi}}}_{m} z = mz$$

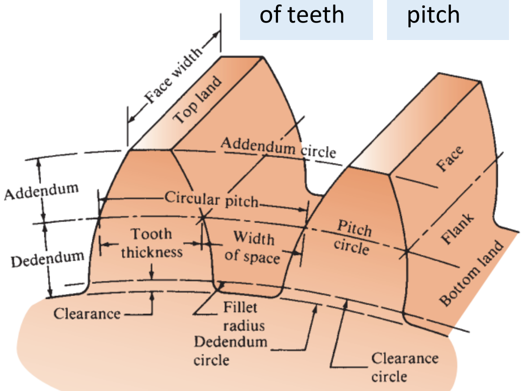


Fig. [Mott 2018]

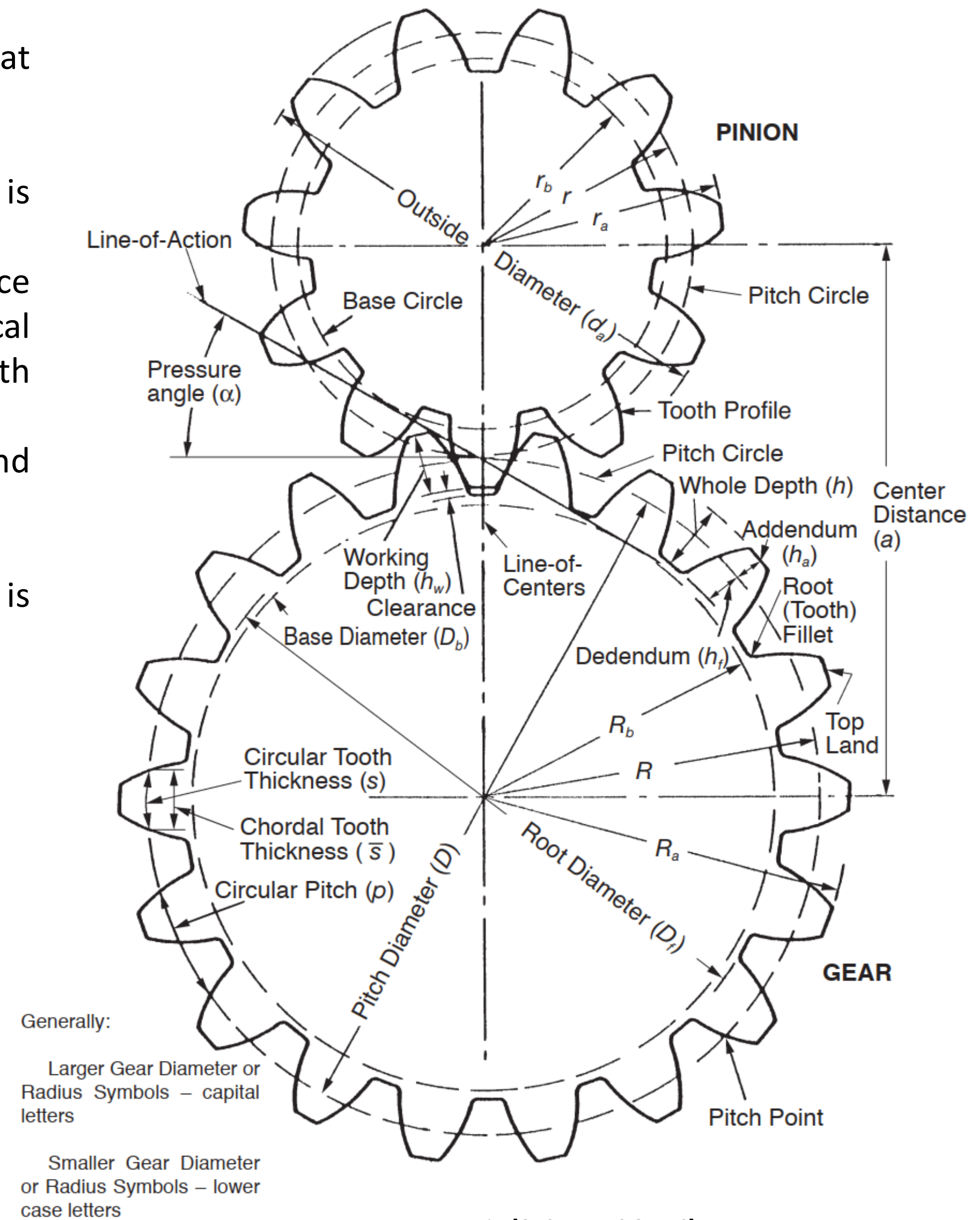


Fig. [QTC METRIC GEARS]

Spur gear terminology (involute profile)

Circular pitch $p_c = \frac{\pi d}{z} = \pi m$ [mm] – arc length measured on pitch diameter between two adjacent teeth profiles.

Values of **module** are normalized and available in standards like ISO. In the case of halical gears values in table concern normal module m_n.

Addendum h_a [mm] – height of tooth between pitch diameter and top land (outside diameter)

$$h_a = ym$$

y – coefficient of teeth height. For normal teeth y = 1

Dedendum h_f [mm] – height of tooth between pitch diameter and bottom land (root diameter)

$$h_f = (y + c^*)m$$

c^* – top clearance coefficient 0,1÷0,35, usually 0,25

$c = mc^*$ [mm] – top clearance

For tooth without profile shift, normal height y = 1 and c^*= 0,25

$$h = h_a + h_f = m(2y + c^*) = 2{,}25m\ [mm]$$ – **total height of tooth**

I	II	I	II	I	II	I	II
0.1		1			5.5	25	
	0.15		1.125	6			28
0.2		1.25			(6.5)	32	
	0.25		1.375		7		36
0.3		1.5		8		40	
	0.35		1.75		9		45
0.4		2		10		50	
	0.45		2.25		11		
0.5		2.5		12			
	0.55		2.75		14		
0.6		3		16			
	0.7		3.5		18		
	0.75	4		20			
0.8			4.5		22		
	0.9	5					

Table - Standard values of module for cylindrical gears [https://www.kggear.co.jp]

Values in columns I are preferable

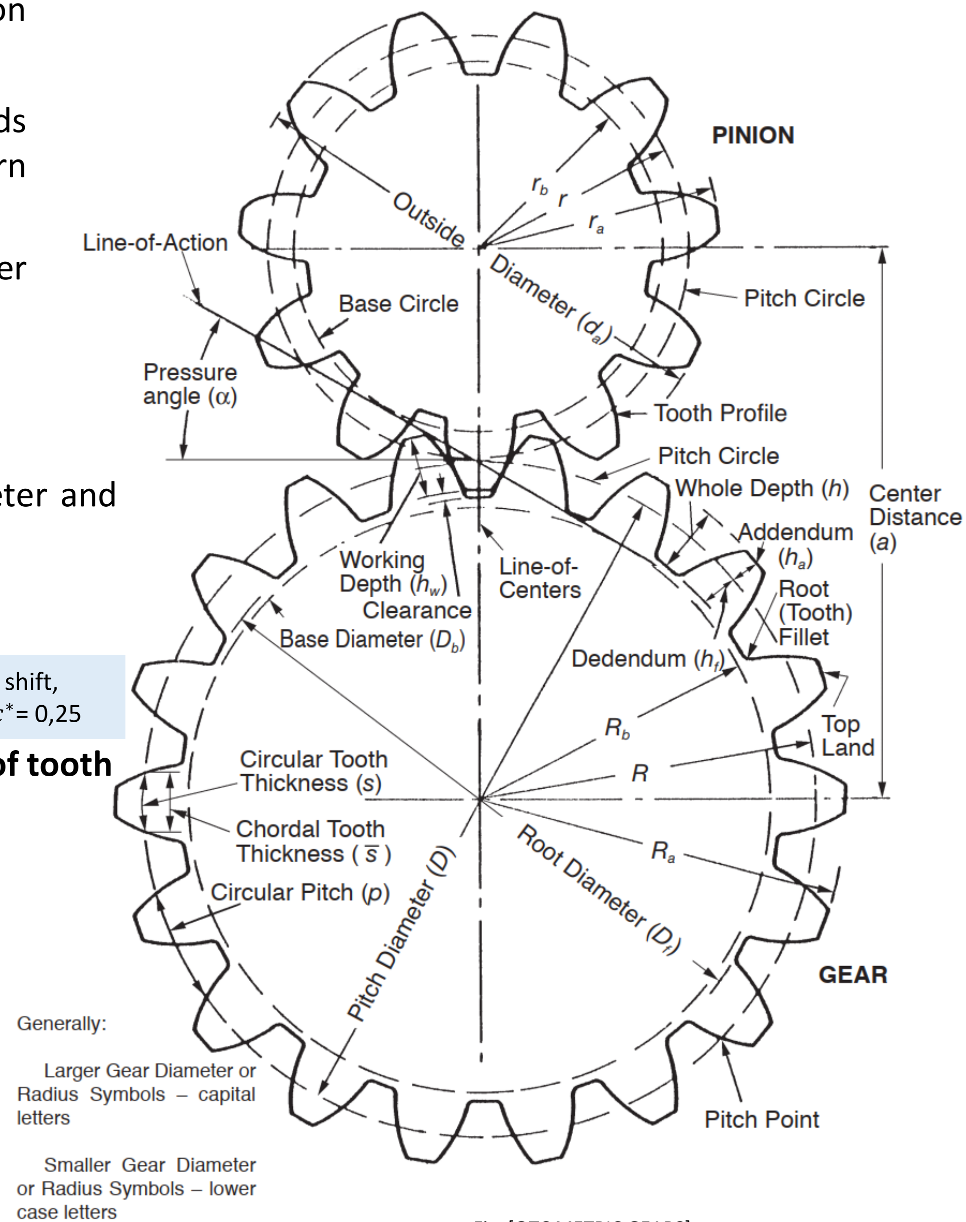


Fig. [QTC METRIC GEARS]

Spur gear terminology (involute profile)

Root diameter d_f [mm]

- pinion

$$d_{f1} = d_1 - 2h_{f1} = m(z_1 - 2y + 2x_1 - 2c^*)$$

- gear

$$d_{f2} = d_2 - 2h_{f2} = m(z_2 - 2y + 2x_2 - 2c^*)$$

Outside diameter d_a [mm]

- pinion

$$d_{a1} = d_1 + 2h_{a1} = m(z_1 + 2y + 2x_1 - 2k)$$

- gear

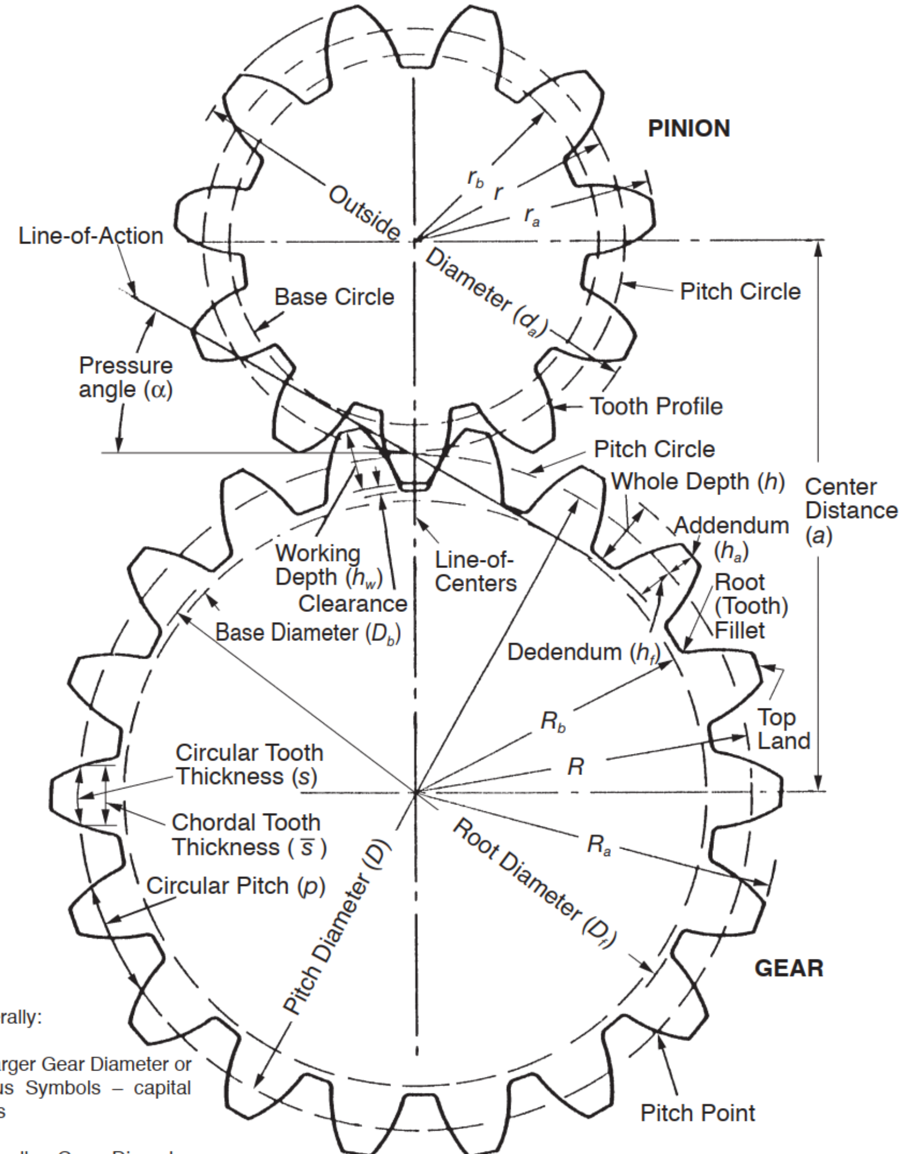
$$d_{a2} = d_2 + 2h_{a2} = m(z_2 + 2y + 2x_2 - 2k)$$

k, x_1, x_2 - equal 0 for profiles withought shifting (profile correction)

Base diameter d_b [mm]

$d_b = d\cos(\alpha)$ – it is a diameter of circle from which the involute is created

α - pressure angle



Generally:

Larger Gear Diameter or Radius Symbols – capital letters

Smaller Gear Diameter or Radius Symbols – lower case letters

Fig. [QTC METRIC GEARS]

Spur gear terminology (involute profile)

Working (operating) pressure angle $\boldsymbol{\alpha_w} = \cos^{-1}\left(\frac{a\cos\alpha}{a_w}\right)$ [°] - it is an angle between tangent line to base circles of gears and line perpendicular to line of centers.

This angle is influenced by pressure angle of tool used to produce teeth and working center distance. It is used to calcualte contact ratio, components of normal force and real distance between gears.

Pressure angle $\boldsymbol{\alpha}$ is always measured on pitch diameter (angle has variable value dependent on the position on involute profile) between tangent direction to the tooth profile and radial direction that is passing the center of basic circle.

Pressure angle of teeth and presure angle of tool used in production are identical in theory. Standard value of pressure angle α is 20°. Historically 14,5° angle was used. Others values that can be applied: 17,5; 22,5°; 25°

Invloute gears must have the same pressure angle to work together (there are more requirements e.g. module).

Pressure angle is important because have influence on: contact ratio, bending and contact stress, efficiency, slipping velocity and minimal number of teeth that could be made without undercutting.

Working pressure angle and pressure angle are the same if working center distance is equal (nominal) center distance.

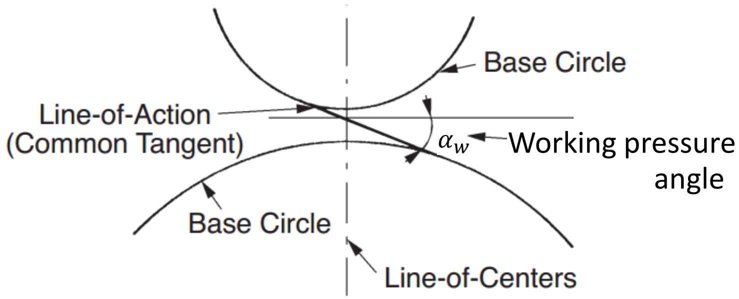


Fig. Working pressure angle α_w, based on: [QTC METRIC GEARS]

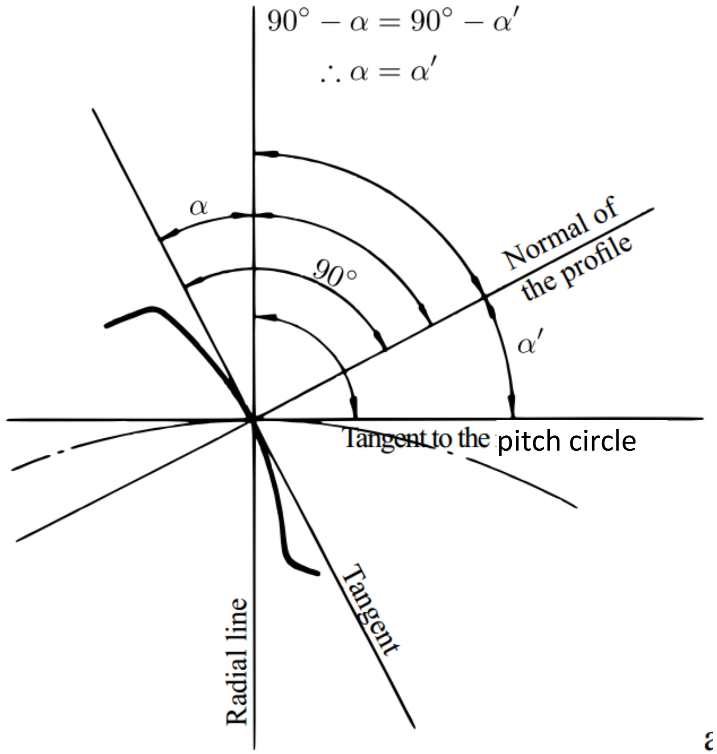


Fig. Pressure angle α, based on: [KHK]

Spur gear terminology (involute profile)

(Nominal) Center distance $a = \frac{(z_1+z_2)m}{2}$ [mm] – it is distance between axes of rotation of pinion and gear when pitch circles of both gears are tangent. With assumption that pitch circles are associated with gears, they roll without sliding during gear rotation.

Working (operating) center distance $a_w = a\frac{\cos\alpha}{\cos\alpha_w}$ [mm] – it is distance between axes of rotation of pinion and gear when working (operating) pitch circles of both gears are tangent. With assumption that working pitch circles are associated with gears, they roll without sliding during gear rotation.

Center distance and working center distance are the same if gears are without profile shift or coefficeints of profile shift are equal $x_1 = -x_2$.

Working (operating) pitch diameters $d_{w1} = 2a_w\frac{z_1}{z_1+z_2}$, $d_{w2} = 2a_w\frac{z_2}{z_1+z_2}$ [mm] – they are the abstract diameters of circles that are always tangent (in point contact) and roll without sliding during gear rotation.

Pitch diamater and working pitch diameter are the same if gears are without profile shift or coefficeints of profile shift are equal $x_1 = -x_2$.

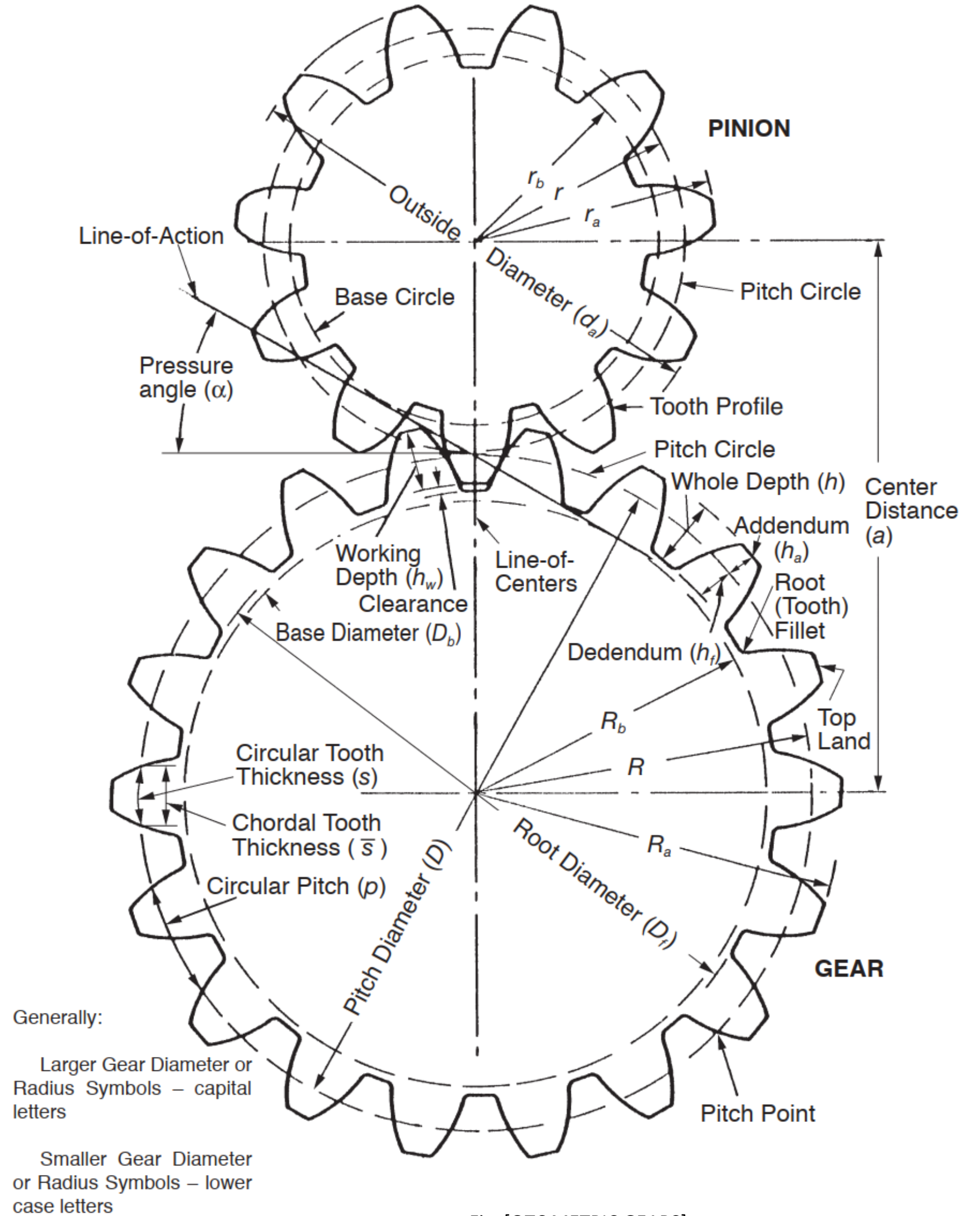


Fig. [QTC METRIC GEARS]

Spur gear terminology (involute profile)

Contact ratio ε_α is a parameter that determine how many pairs of teeth are in contact on average during motion. It is defined as a ratio of the length of contact (action) *AE* to the base pitch p_b. In typical application contact ratio should be at least 1,2

$$\varepsilon_\alpha = \frac{AE}{p_b}$$

The difference between base pitch and circular pitch is that is measured on base diameter.

$$p_b = \frac{\pi d_b}{z} = \frac{2\pi r_b}{z}$$

Knowing that:

$$d_b = dcos(\alpha)$$

thus $$p_b = \frac{\pi d \cos\alpha}{z} = \pi m \cos\alpha = p_c \cos\alpha$$

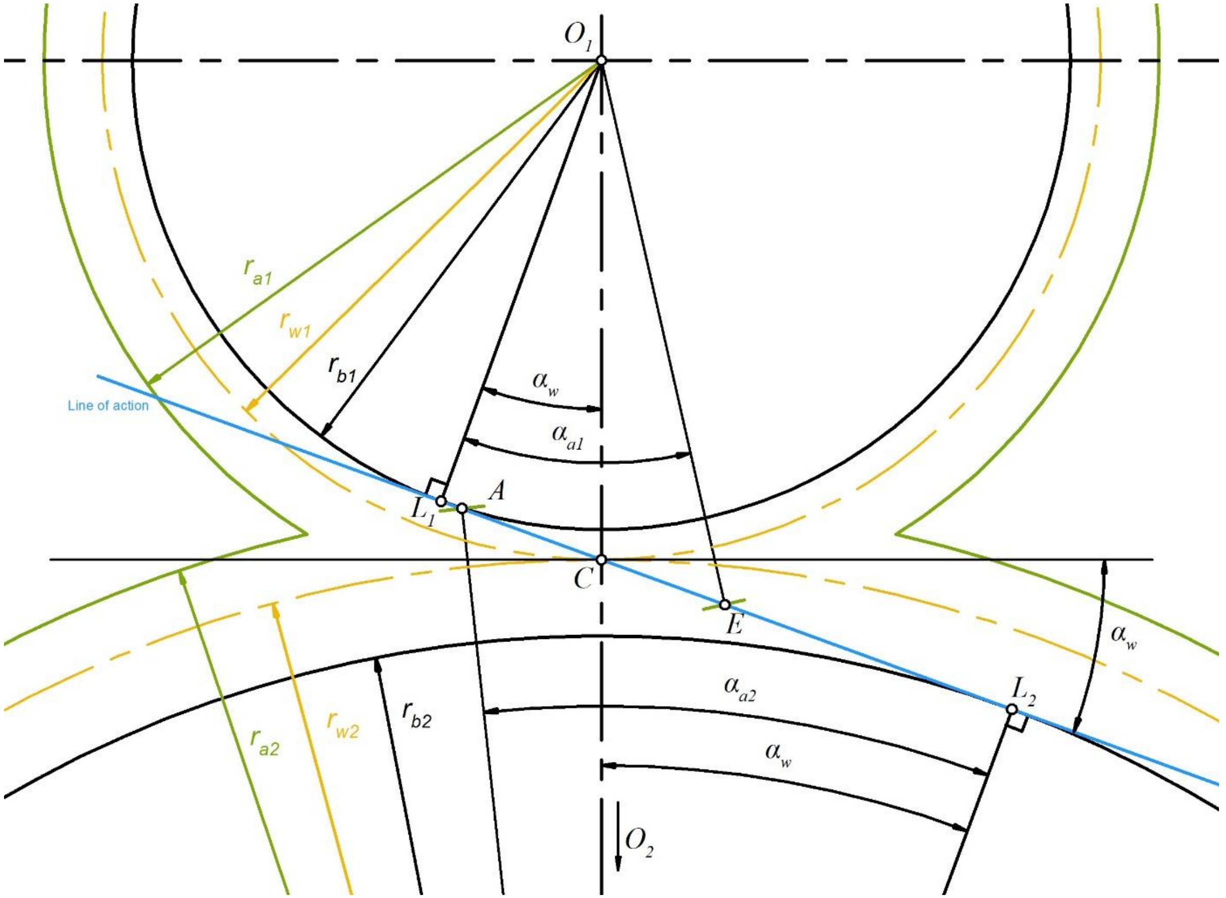


Fig. Contact ratio

Spur gear terminology (involute profile)

Contact ratio ε_α

Length of interval *AE* can be exppressed as

$$AE = L_1E + L_2A - L_1L_2$$

where

$L_1E = r_{b1}\tan\alpha_{a1}$
$L_2A = r_{b2}\tan\alpha_{a2}$
$L_1L_2 = r_{b1}\tan\alpha_w + r_{b2}\tan\alpha_w$
$= (r_{b1} + r_{b2})\tan\alpha_w$

Pressure angle of tooth tip α_a

$$\cos\alpha_a = \frac{d_b}{d_a} \rightarrow \alpha_a = \cos^{-1}\frac{d_b}{d_a}$$

$$\cos\alpha_w = \frac{a}{a_w}\cos\alpha \rightarrow \alpha_w = \cos^{-1}\frac{a}{a_w}\cos\alpha$$

α_w- working pressure angle, if $x_1$and x_2 are equal 0 or $x_1 = -x_2$, $\alpha_w = \alpha$

x_1, x_2 - coefficients of profile shift (correction)

a_w- working centre distance, if $x_1$and x_2 are equal 0 or $x_1 = -x_2$, $a_w = a$

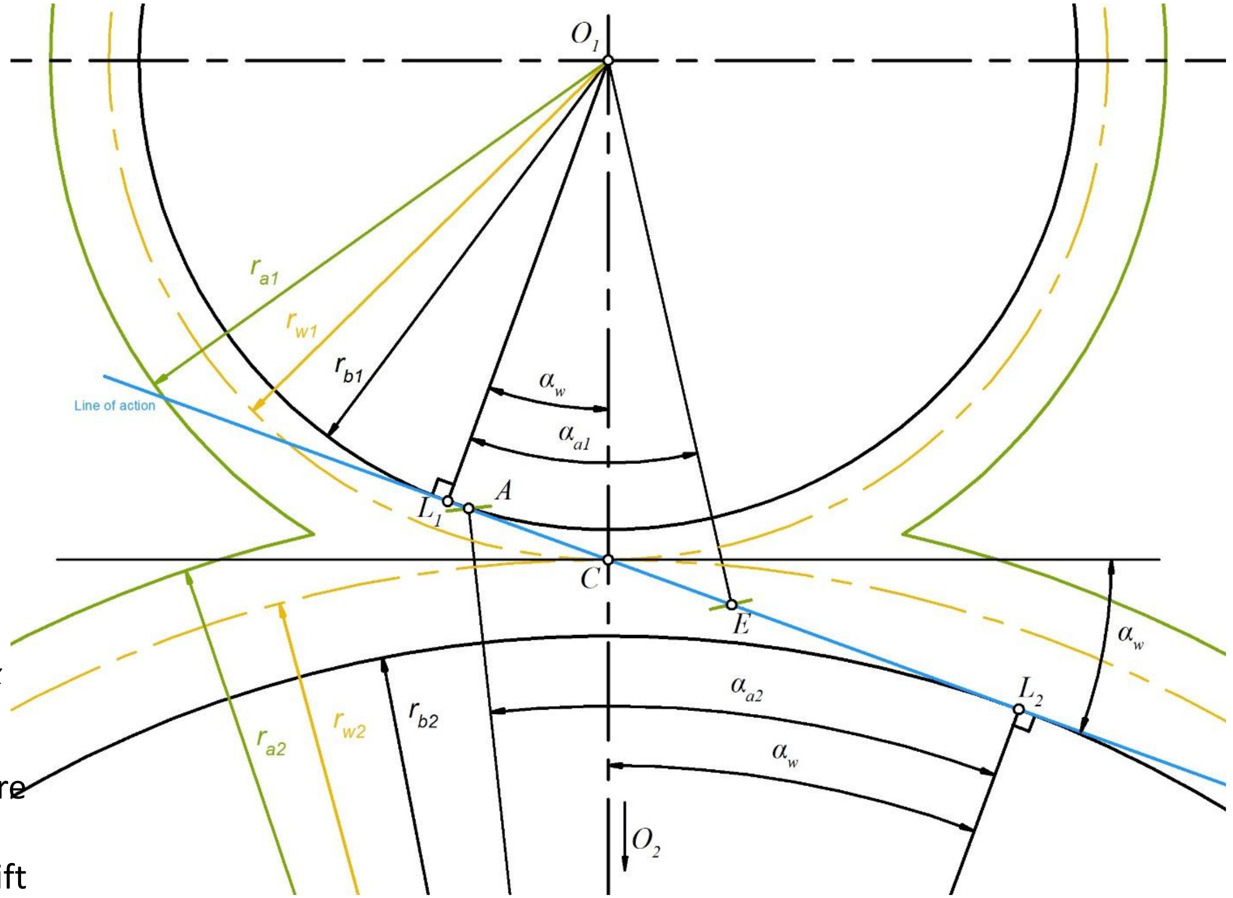


Fig. Contact ratio

$$\varepsilon_\alpha = \frac{AE}{p_b} = \frac{r_{b1}\tan\alpha_{a1} + r_{b2}\tan\alpha_{a2} - (r_{b1} + r_{b2})\tan\alpha_w}{\frac{2\pi r_b}{z}}$$

$$\varepsilon_\alpha = \frac{z[r_{b1}(\tan\alpha_{a1} - \tan\alpha_w) + r_{b2}(\tan\alpha_{a2} - \tan\alpha_w)]}{2\pi r_b} = \boxed{\frac{1}{2\pi}[z_1(\tan\alpha_{a1} - \tan\alpha_w) + z_2(\tan\alpha_{a2} - \tan\alpha_w)]}$$

Spur gear terminology (involute profile)

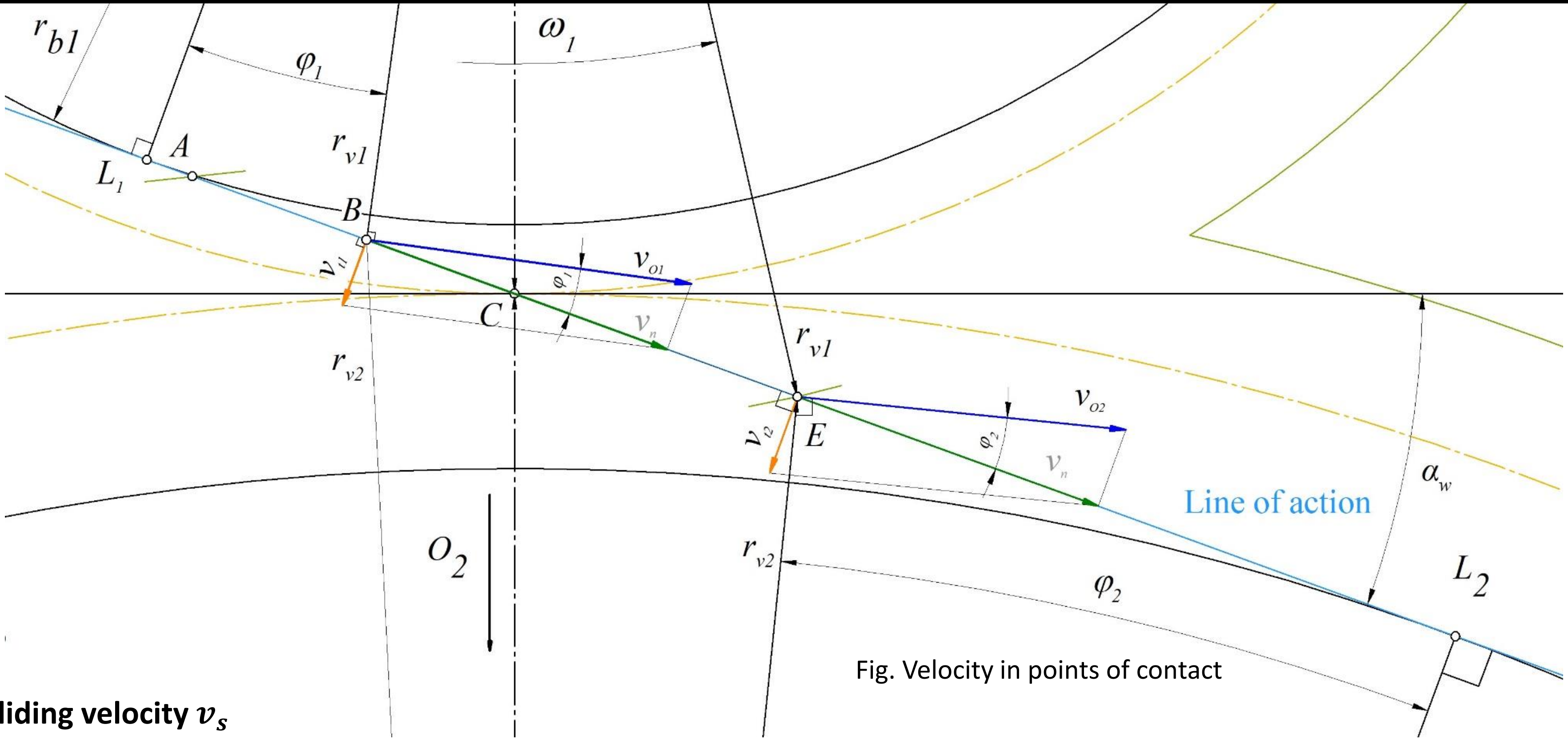


Fig. Velocity in points of contact

Sliding velocity v_s

Relative motion between teeth surface beeing in contact have influence on teeth wear, gear unit efficiency, teeth temperature and lubrication condition. Relative motion is determined by sliding velocity.

Theoretical contact point progress along line of action. Velocity of this point $\vec{v}_n$ is the sum of (lifting) velocity $\vec{v}_{01}$ resulting from rotational movement of gear and tangential (relative) velocity $\vec{v}_{t1}$ which is defined by involute profile. According to Fig., where are present two contact point *B* and *E*, normal velocity along LOA is equal:

$$\vec{v}_n = \vec{v}_{01} + \vec{v}_{t1} = \vec{v}_{02} + \vec{v}_{t2}$$
$$v_n = r_{b1}\omega_1 = r_{b2}\omega_2$$

Spur gear terminology (involute profile)

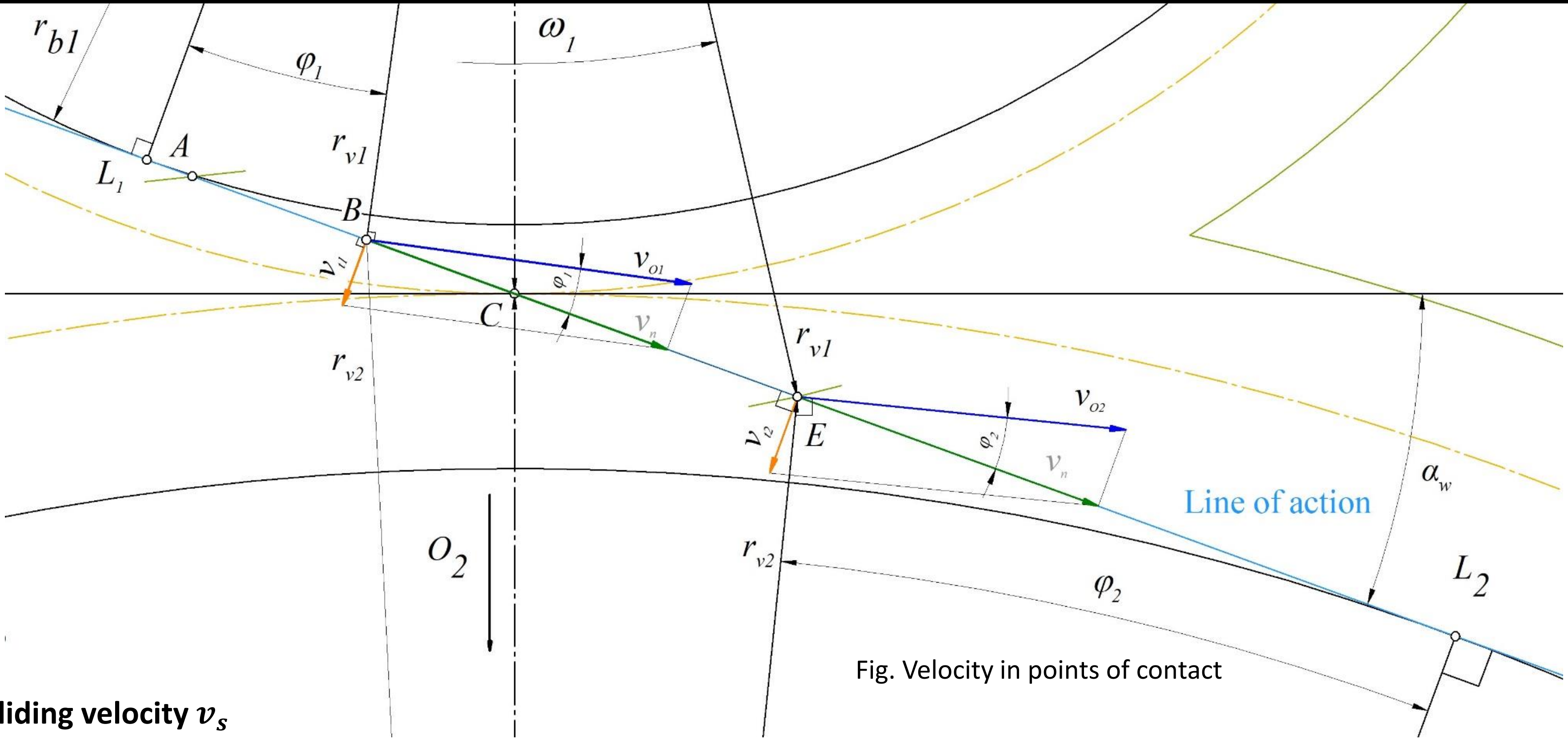


Fig. Velocity in points of contact

Sliding velocity v_s

Normal velocities v_n of both gears are equal. This is due to assumption that teeth are always in contact and property of involute profile which assure constant gear ratio (velocity or speed ratio).

$v_{O1} = r_{v1}\omega_1$ where: $r_{v1} = r_{b1}/\cos\varphi_1$

$v_{O2} = r_{v2}\omega_2$ where: $r_{v2} = r_{b2}/\cos\varphi_2$

$v_{t1} = v_n \tan\varphi_1$

$v_{t2} = v_n \tan\varphi_2$

Spur gear terminology (involute profile)

Sliding velocity v_s

The value of sliding velocity for the pinion is equal
$v_{s1} = v_{t1} - v_{t2}$

and for the wheel
$v_{s2} = v_{t2} - v_{t1}$

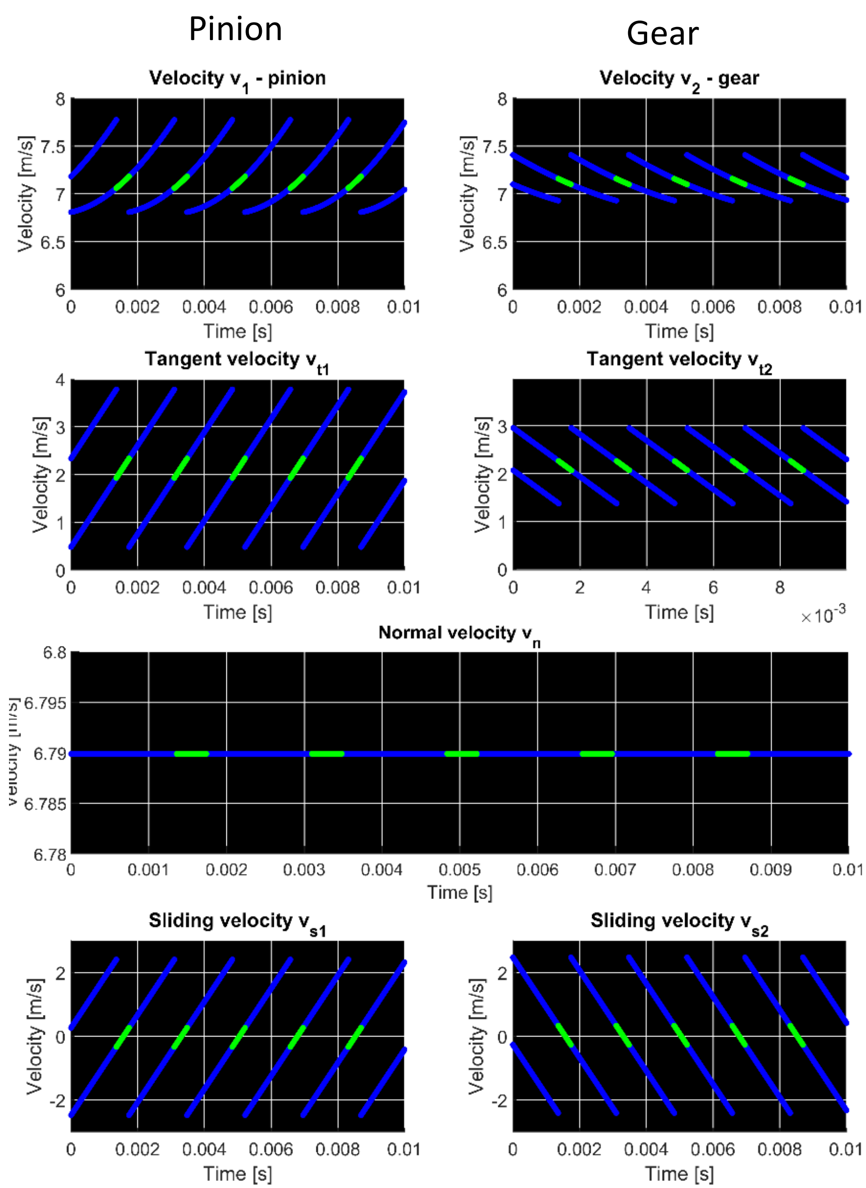


Fig. Example resaults [Jedliński 2021]

Spur gear terminology (involute profile)

Top clearance and **backlash** are necessary for proper work of gears. Parts are made with some tolerance and without backlash and clearance teeth can jamming and conditions for lubrication are poor. Even then entire gear unit was made as ideal, parts are displace and deform (strain) because of forces (static and dynamic) and thermal expansion.

Backlash should be not excessive especially then direction of rotation is changed during exploitation. Value of backlash is determined based on production precision and working conditions.

Backlash generally can be introduced in two ways: during machining by closing a cutting tool to gear(s) or by changing center distance.

From practical point of view there are situations when it is more conveniet to measure **linear backlash** $\boldsymbol{j}$ using indicator instead of measure **normal backlash** $\boldsymbol{j_n}$
The dependency bettwen this two backlashes is $j = \frac{j_n}{\cos \alpha}$

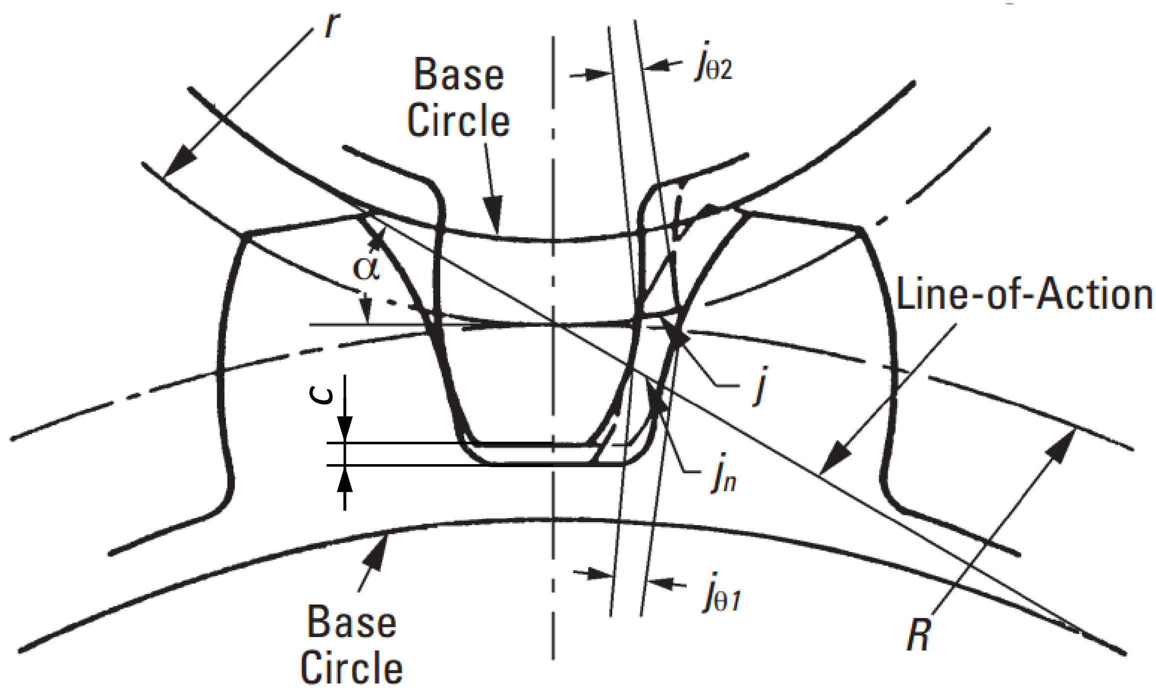


Fig. Backlash and clearance between teeth[QTC METRIC GEARS]

Top clearance $c = c^* m$
Normal backlash j_n - it is measured along line of action

$$j_n = 2\Delta a \sin \alpha$$

Δa – difference between theoretical center distance and actual
Linear backlash j – it is measured on pitch circle

$$j = 2\Delta a \tan \alpha$$

Helical gear - introduction (involute profile)

Helical gears are similar to spur gears in the sense that teeth are created on cylinder and depending on the method the same machines and tools can be use for both types of gears. Line of tooth* in spur gears is a line and in helical gears is a helix.

Fig. Hobbing spur gear [https://en.wikipedia.org/wiki/Hobbing]

Fig. Hobbing helical gear [http://vestavia.eu/product/manufacturing-helical-gears/]

**Line of tooth* – line or curve created by intersection of pitch cylinder or cone and tooth working surface

Helical gear - introduction (involute profile)

Helical gears in compare to spur gears have (due to helix angle > 0) higher contact ratio and change between min. and max. value of this parameter is less rapid. This positive features implies advantages:

- lower level of vibrations and noise,
- higher load capacity,
- min. number of teeth withouth undercut is smaller.

But skew teeth are the cause of:

- axial force,
- possibly higher friction that could reduce efficiency and increase wear.

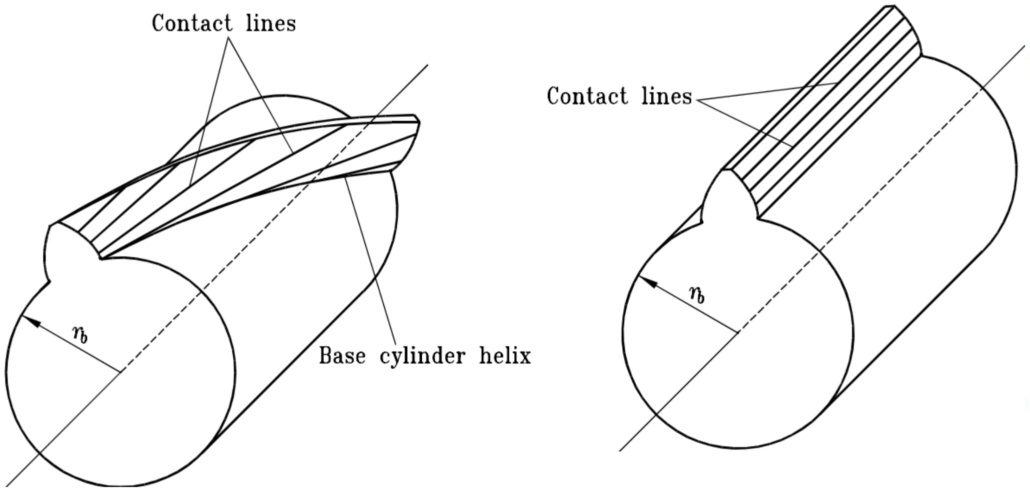


Fig. Contact lines of spur and helical gears during mesh [Litvin 2004]

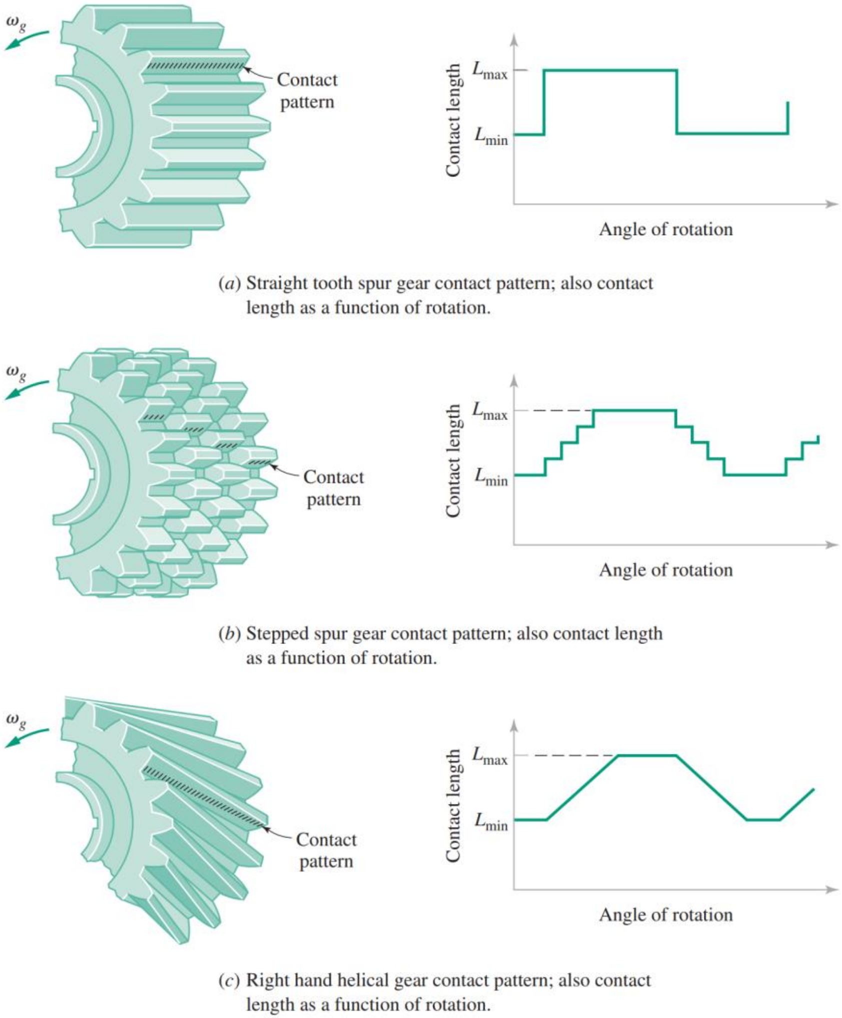


Fig. Contact pattern of different gears and diagrams with contact length progress [Collins 2010]

Helical gear - introduction (involute profile)

Most often helical gears are working in paralel shafts. Helical angle on both gears is the same but have different direction left and right. If helical angle is the same and have the same direction shafts are at 90 ° angle and this type of gear is named crossed helical or screw gears. It is possible to combine two gears with different value of helical angle and the same direction to receive non standard angle between shafts.

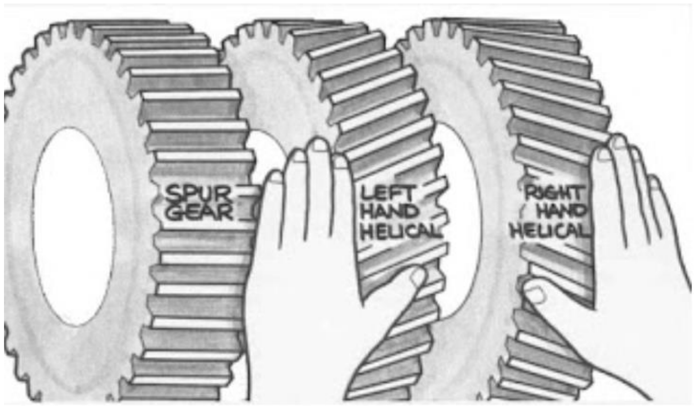


Fig. [https://www.hkdivedi.com/2015/12/basic-of-helical-gear.html]

(*a*) Right hand and left hand helical gears with 20° helix angle and parallel axes

(*b*) Two right handed helical gears with 45° helix angle and crossed axes

Fig. Combination of helical gears with the same helical angle a) left-hand and right-hand, b) two right-hand [Litvin 2004]

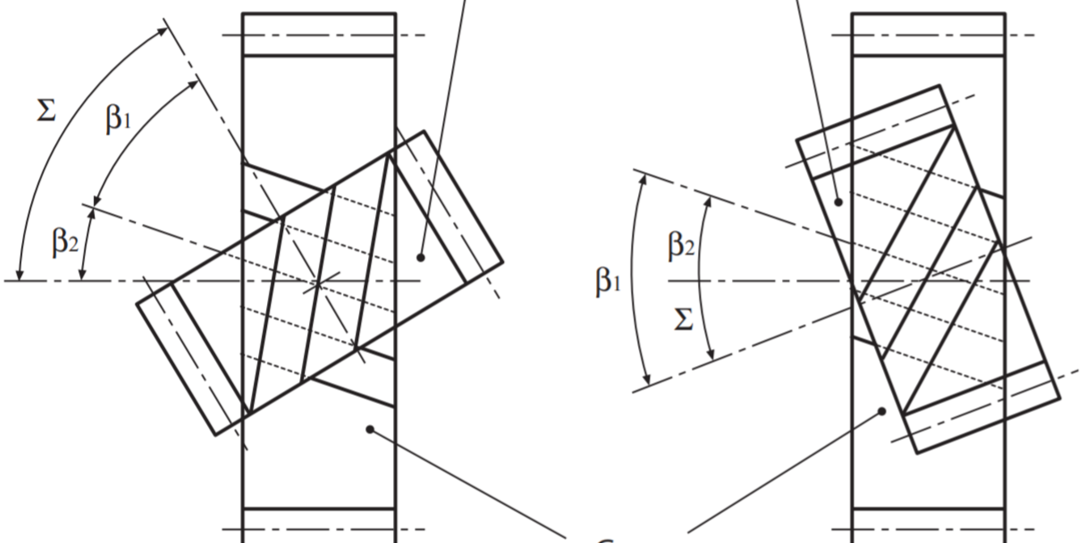


Fig. Combination of helical gears with the same direction (left-hand or right-hand) and different helix angle [KYOUIKU]

Helical gear terminology (involute profile)

Helix angle β if most often defined or state on pitch cylinder.
There is a plane (green color) tangent to pitch cylinder and tangent to helix. Helix and tangent plane have a common point*. Helix is projected into tangent plane and it is seen as a green line. Axis of gear is also projected on tangent plane and it is seen as an orange line. Helix angle β is between helix line projection and gear axis projection.

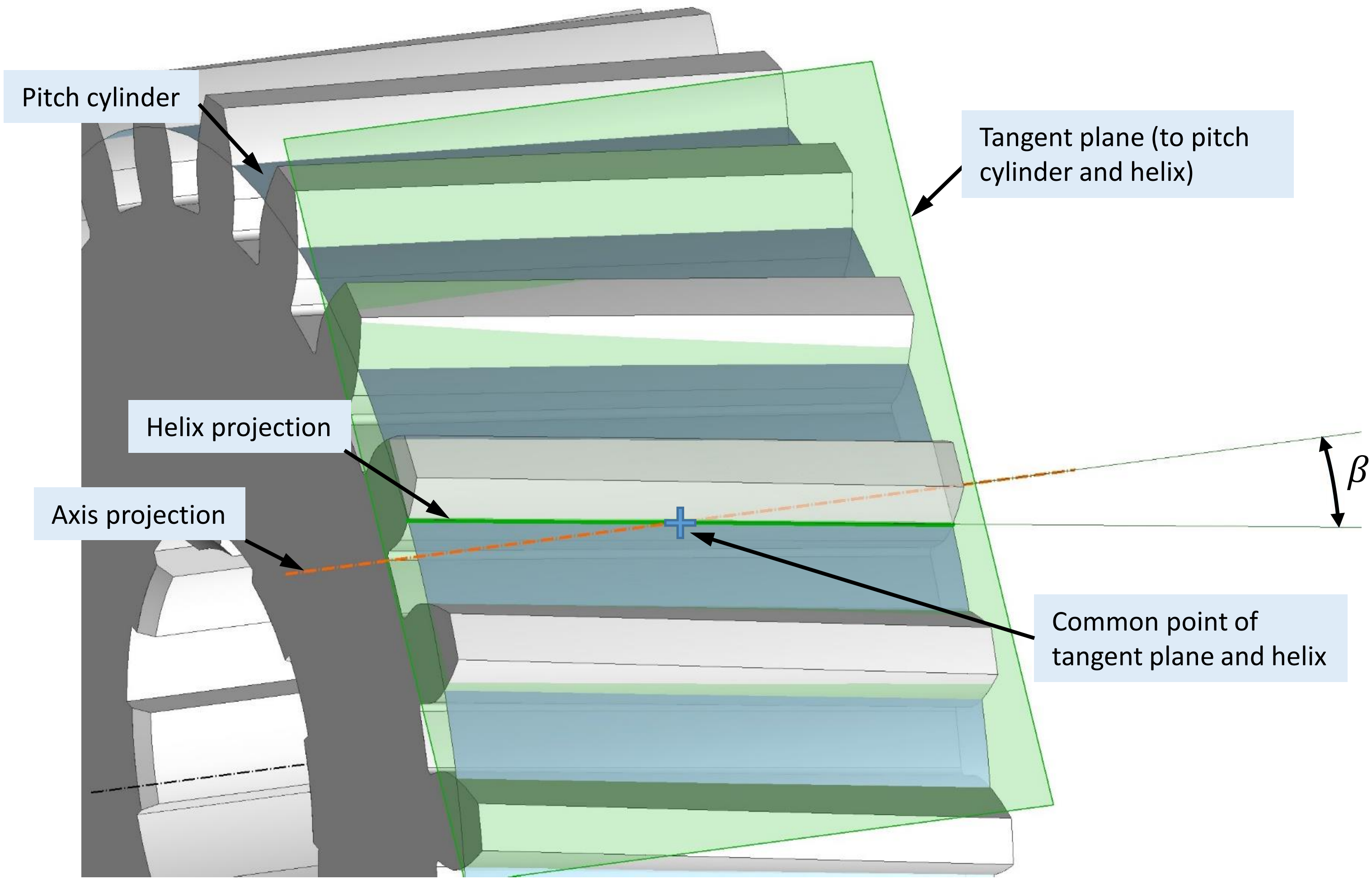


Fig. Helix angle defined on pitch cylinder

* The position of a point is dependent on position of tangent plane, but this do not have influence on value of helix angle).

Helical gear terminology (involute profile)

Standard value of helix angle is form about 8° to 25°. For double helical (herringbone gears) helix angle can have value up to 45°.
Value of helix angle is dependent on diameter of cylinder thus helix angle e.g. on base cylinder have different (smaller) value than on pitch cylinder.

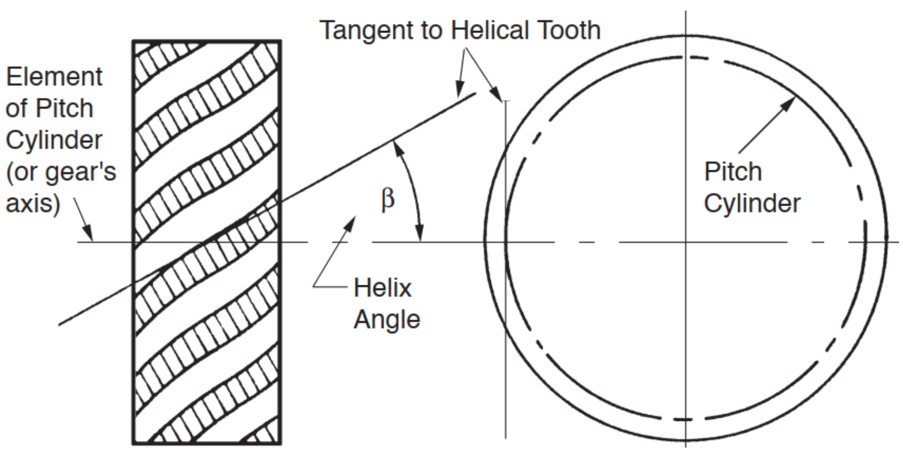


Fig. Creation of involute profile in helical gear [QTC METRIC GEARS]

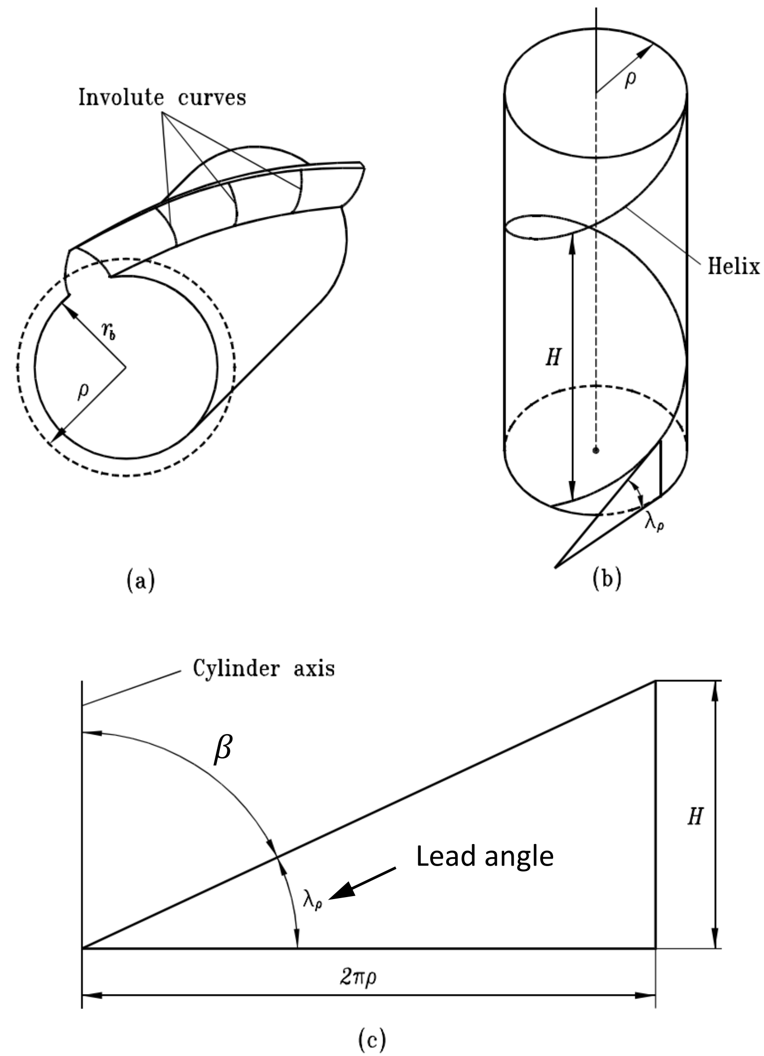


Fig. [Litvin 2004]

Helical gear - introduction (involute profile)

Usually the properties of helical gears are explained with assumption that teeth are made by rack (Maag) tool. In spur gears rack is making reciprocating motion parallel to axis of gear rotation. To make helical teeth tool is making reciprocating motion along line that is at helix angle to axis of gear rotation. If this angle is 0° it is received spur gear.
In helical gears dimensions of cutter are obtained on teeth in normal plane (plane perpendicular to line of tooth), but involute profile of teeth is in transverse plane (perpendicular to gear axis) because in this plane gear and cutter are cooperating. Calculation concerning profile of teeth are done in transverse plane and that connected to the cutting tool usually in normal plane. *The height of teeth are the same in any plane.*

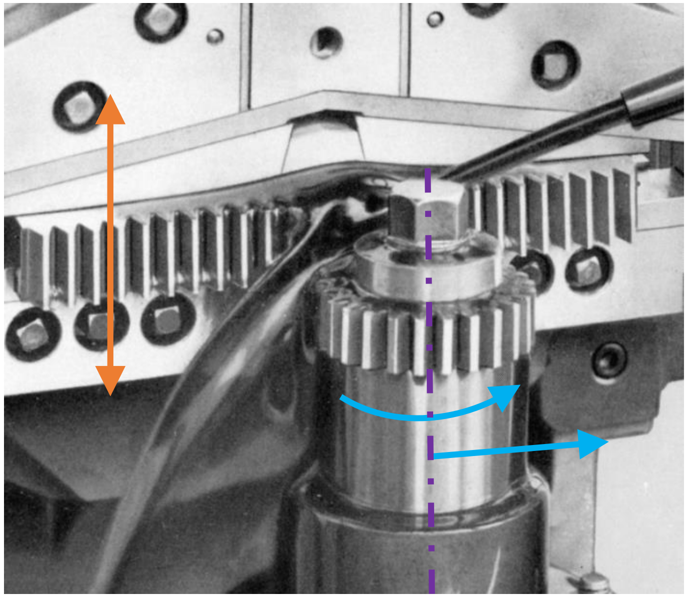


Fig. Cutting spur gear [http://www.lathes.co.uk/maag/]

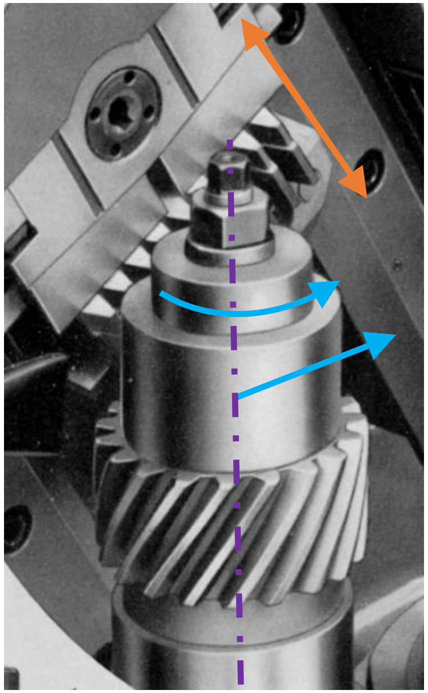


Fig. Cutting helical gear [http://www.lathes.co.uk/maag/]

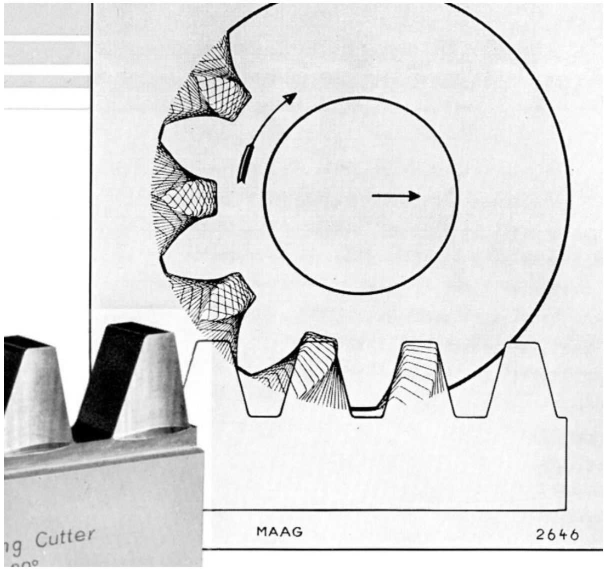


Fig. Cutting spur gear [http://www.lathes.co.uk/maag/]

Helical gear - introduction (involute profile)

Working surface of tooth with involute profile in helical gear can be obtain in the way presented on below Figure. It can be imagine that there is an unrolled tape on the base cylinder. On this tape on flat (plannar) part is drawn a line AB which is not parallel to axis of cylinder (is spur gear is parallel). Tape is rolled up in the way that part not connected with cylinder is flat and tangential to it. Then line AB change its position to A_oB_o. The path of line from start to end positions create working tooth surface which have involute profile in the plane (transverse) parepndicular to axis of cylinder. A_oB_o on cylinder is a helix (curve).

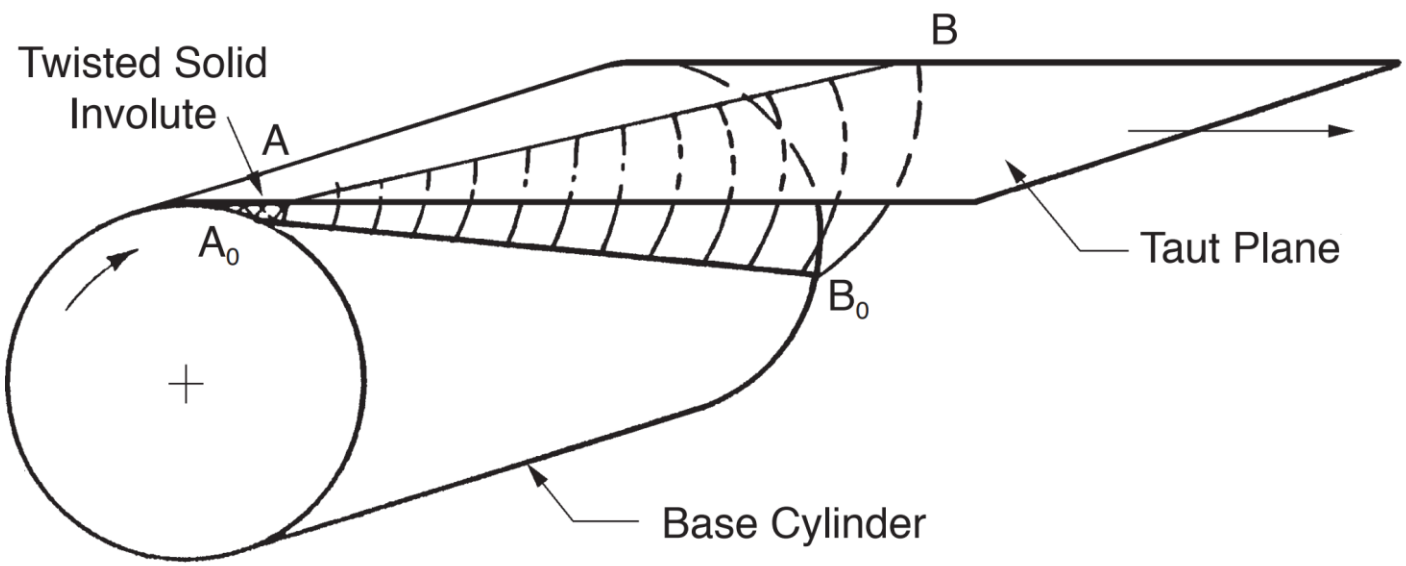


Fig. Creation of involute profile in helical gear [QTC METRIC GEARS]

Helical gear terminology (involute profile)

Normal and transverse planes and basic parameters of teeth are presented on the example of rack (wheel that have diameter equal infinity). As was mentioned before parameters of teeth depended on situation should be known in normal or transverse plane. Parameters in normal plane *N-N* (perpendicular to line of tooth) have subscript *n*, in transverse plane T-T (perpendicular to gear axis) have subscript *t*. Sometimes it is used third plane named axial plane (passes through gear axis) and parameters have subscript *x*.

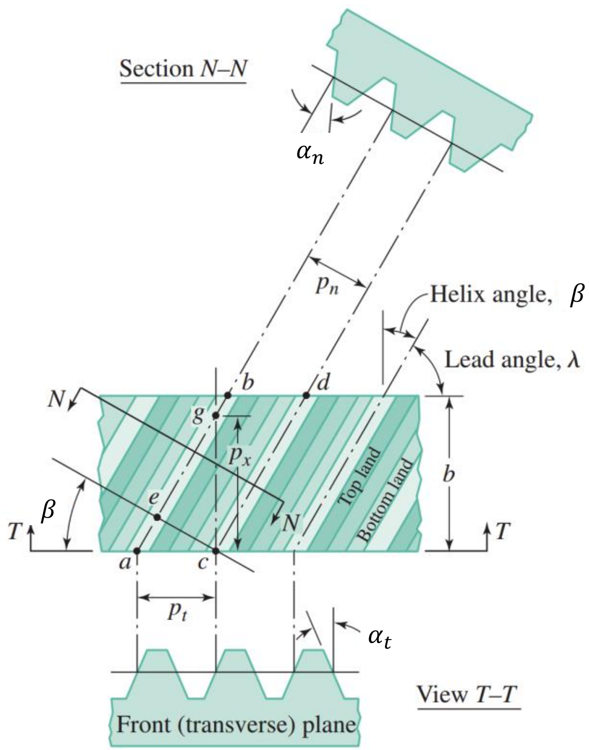


Fig. Creation of involute profile in helical gear [QTC METRIC GEARS]

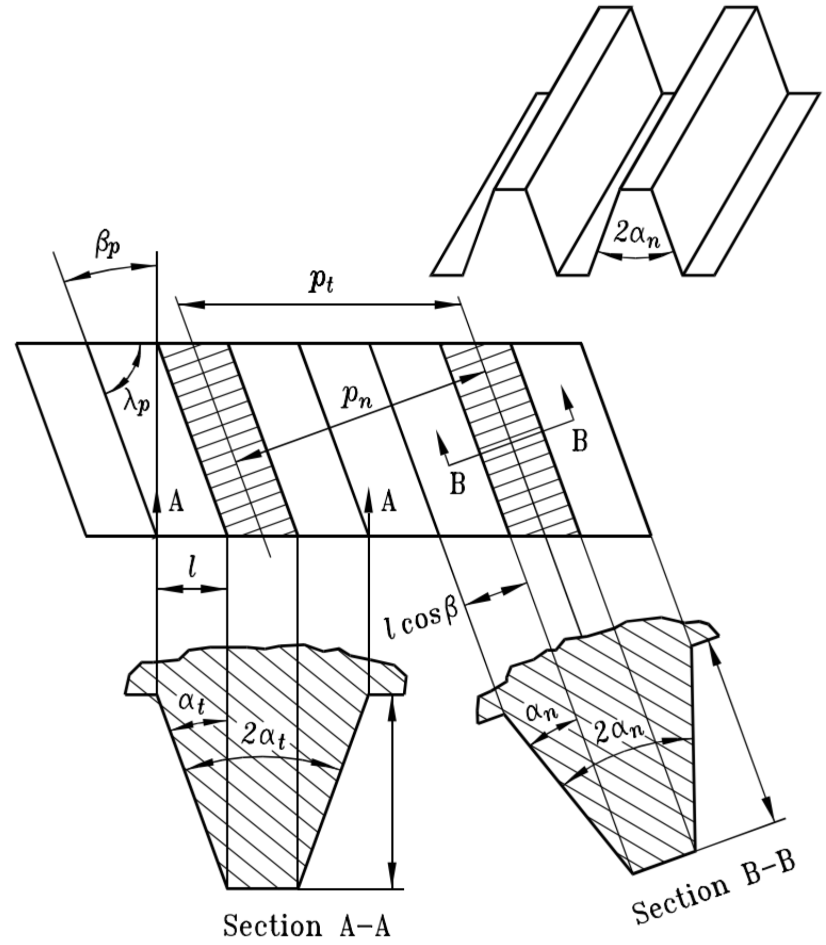


Fig. [Litvin 2004]

Helical gear terminology (involute profile)

Definitions of parameters of teeth are the same as for spur gears, moreover value in normal plane are the same as for spur gear e.g. $m_n = m$, $p_n = p$.
Based on below Figure relation of parameters in normal and tranverse plane can be esatablish:

Transverse circular pitch $\boldsymbol{p_t}$ [mm] $\quad p_t = \dfrac{p_n}{\cos\beta}$ $\quad$ where $p_n = \pi m_n, p_t = \pi m_t$

and in analogical way other

Transverse module $\boldsymbol{m_t}$ [mm] $\quad m_t = \dfrac{m_n}{\cos\beta}$

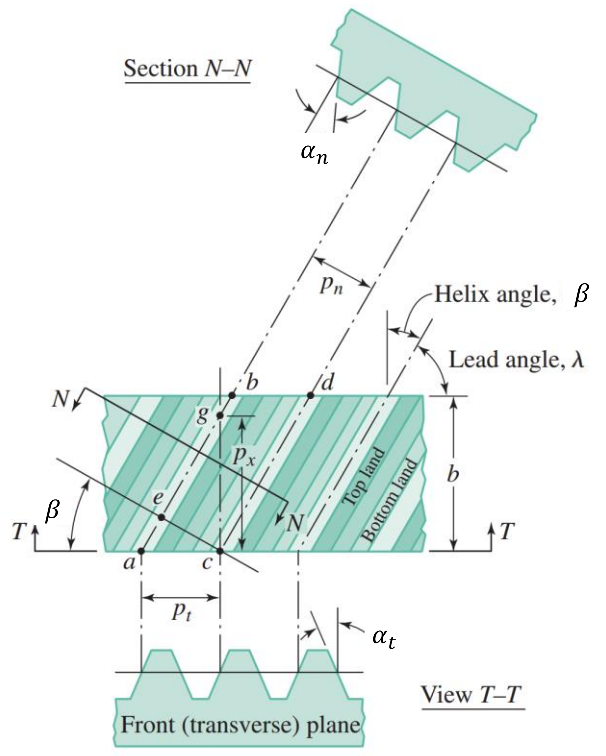


Fig. Creation of involute profile in helical gear [QTC METRIC GEARS]

Addendum $\boldsymbol{h_a}$ [mm] $\quad h_a = y_t m_t = y_n m_n$

$$y_t = y_n \cos\beta$$

Dedendum $\boldsymbol{h_f}$ [mm] $\quad h_f = (y_t + c_t^*) m_t = (y_n + c_n^*) m_n$

$c_t^* = c_n^* \cos\beta$ – top clearance coefficient
$c = c_t^* m_t = c_n^* m_n$ [mm] – top clearance

For tooth without profile shift, normal height y = 1 and c^*= 0,25

Total height of tooth $\boldsymbol{h}$ [mm]

$$h = h_a + h_f = m_t(2y_t + c_t^*) = m_n(2y_n + c_n^*) = 2{,}25 m_n\ [mm]$$

Relationship between angels: tranverse pressure angle α_t, normal pressure angle α_n, axial pressure angle $\alpha_x$and helix angle β

$$\tan\alpha_t = \frac{\tan\alpha_n}{\cos\beta} \qquad \tan\alpha_x = \frac{\tan\alpha_t}{\tan\beta}$$

Helical gear terminology (involute profile)

Pitch diameter *d* [mm]

$$d = zm_t = \frac{zm_n}{\cos\beta}$$

Root diameter d_f [mm]

- pinion

$$d_{f1} = d_1 - 2h_{f1} = m_t(z_1 - 2y_t + 2x_{t1} - 2c_t^*)$$

- gear

$$d_{f2} = d_2 - 2h_{f2} = m_t(z_2 - 2y_t + 2x_{t2} - 2c_t^*)$$

Outside diameter d_a [mm]

- pinion

$$d_{a1} = d_1 + 2h_{a1} = m_t(z_1 + 2y_t + 2x_{t1} - 2k_t)$$

- gear

$$d_{a2} = d_2 + 2h_{a2} = m_t(z_2 + 2y_t + 2x_{t2} - 2k_t)$$

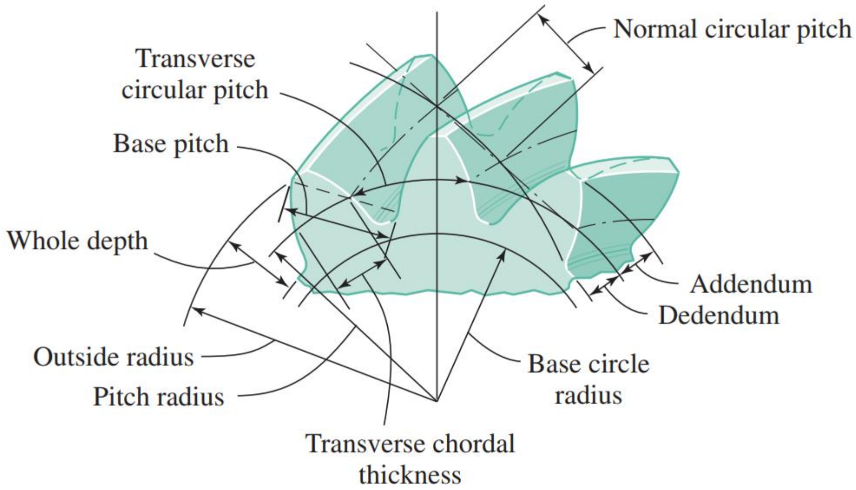


Fig. Basic dimentions of helical gear [Collins 2010]

k_t, x_{t1}, x_{t2} - equal 0 for profiles withought shifting (profile correction)

Base diameter d_b [mm]

$$d_b = d\cos(\alpha_t)$$

Relationship between diameters

$$\frac{d_w}{\tan\beta_w} = \frac{d}{\tan\beta} = \frac{d_b}{\tan\beta_b}$$

Center distance a [mm]

$$a = \frac{(z_1 + z_2)m_t}{2} = \frac{(z_1 + z_2)m_n}{2\cos\beta}$$

Helical gear terminology (involute profile)

Contact ratio ε

Issue of finding contact ratio is divided into two parts. Firstly gears are treated as spur gears and the same formula is used. It should be keep in mind to use parameters for transverse plane. ***Transverse contact ratio ε_α*** is equal:

$$\varepsilon_\alpha = \frac{1}{2\pi}[z_1(\tan\alpha_{a1t} - \tan\alpha_t) + z_2(\tan\alpha_{a2t} - \tan\alpha_t)]$$

Second part include fact that teeth are not straight line but skew that increase time of mesh. ***Face*** (***axial***) ***contact ratio ε_β*** is expressed as:

$$\varepsilon_\beta = \frac{b\tan\beta}{p_t} = \frac{b\sin\beta}{p_n} = \frac{b}{p_x}$$

b – face width (the active part)
$p_x = p_t/\tan\beta -$ axial circular pitch

Contact ratio ε
$\varepsilon = \varepsilon_\alpha + \varepsilon_\beta$

For gears with shifted profiles and $a \neq a_w$
$\alpha_t = \alpha_{wt}$

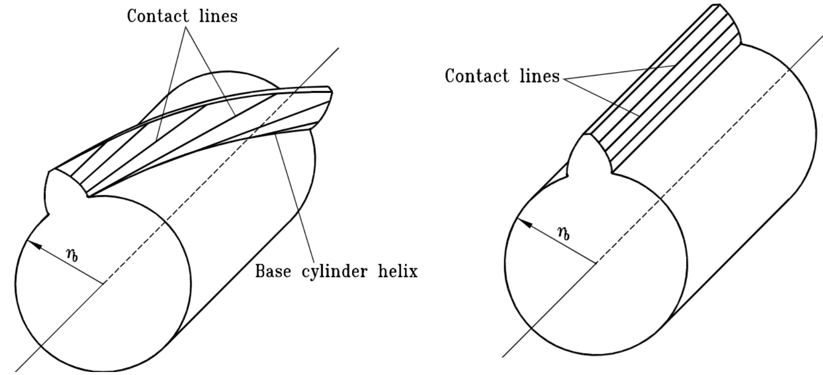


Fig. Contact lines of spur and helical gears during mesh [Litvin 2004]

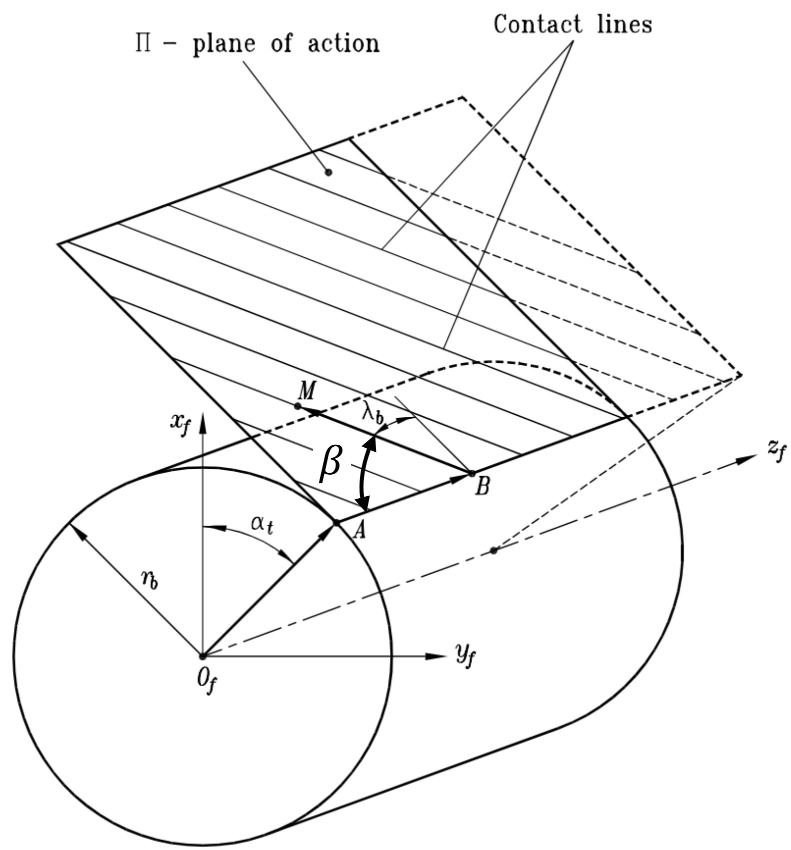


Fig. Plane of action and contact lines in helical gears [Litvin 2004]

Helical gear terminology (involute profile)

Equivalent (virtual) spur gear and equivalent (virtual) number of teeth

There are two main usage of equivalent spur gear and equivalent number of teeth. Having this theoretical (equivalent) gear formulas used to determin strenght of spur gear teeth can be used to helical gear teeth. Second reason is to plan process of manufacturing with cutting tools used to produce spur gears.
Equivalent (vitual) spur gear can be determined for different cylinders like pitch cylinder, base cylinder or working pitch cylinder. For strenght of teeth is used pitch cylinder and for manufacturing base cylinder.

Equivalent (virtual) spur gear defined on pitch cylinder is a gear that have the same strenght as helical gear. It has cylindrical shape with straight teeth.

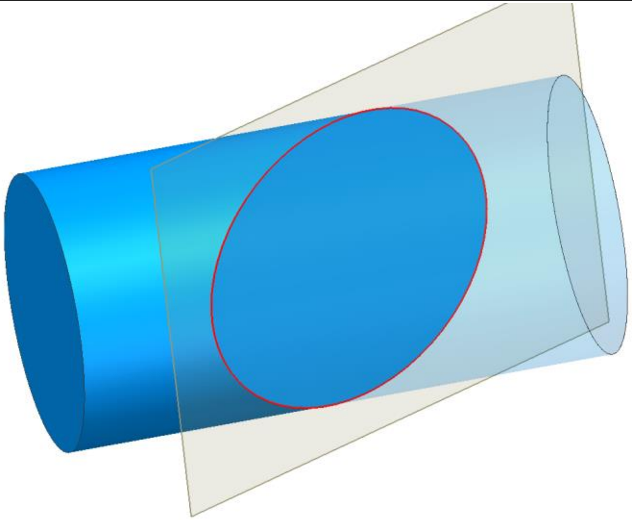


Fig. Ellipse created by cylinder and plane

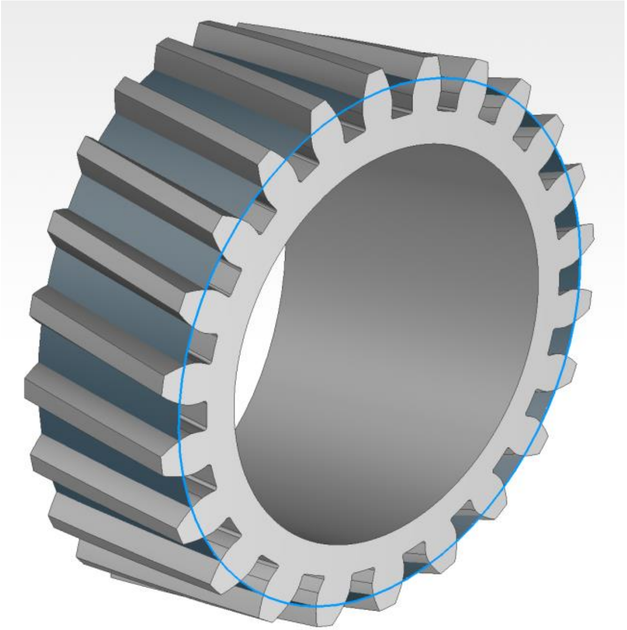


Fig. Helical gear cut by normal plane. Pitch cylinder shown as surface.
Ellipse created by normal plane and pitch cylinder is shown as blue curve

Helical gear terminology (involute profile)

Equivalent (virtual) spur gear and equivalent (virtual) number of teeth

Determination of equivalent spur gear:
1. Cut helical gear by normal plane
Pitch circle have a shape of ellipse in this plane. Semi-minor axis is equal $b = d/2$ and semi-major axis $a = d/(2\cos\beta)$ of this ellipse.

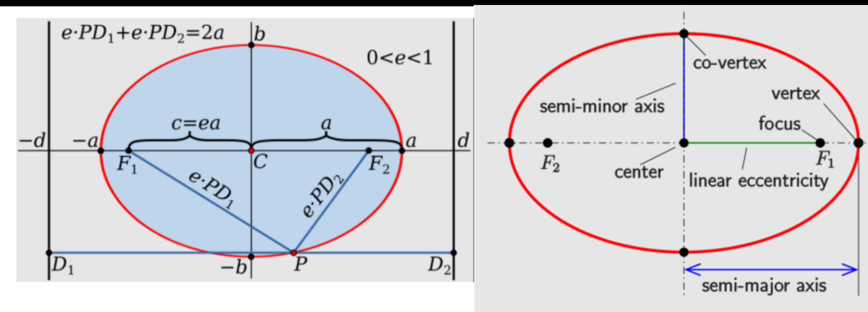
2. Determin pitch circle diameter d_n of equivalent spur gear

Radius of pitch circle of equivalent spur gear r_n is equal radius of curvature of ellipse on semi-minor axis (point *b* in top figure, point *C* – pitch point -in bottom figure).
Based on properties of ellipse radius ρ of curvature at point (vertex) *b* is equal

$$\rho = \frac{a^2}{b} = r_n$$

$$r_n = \frac{\left(\dfrac{d}{2\cos\beta}\right)^2}{\dfrac{d}{2}} = \frac{d}{2\cos^2\beta}$$

$$d_n = \frac{d}{\cos^2\beta}$$



Figs. Main dimentions of ellipse [www.wikipedia pl and en]

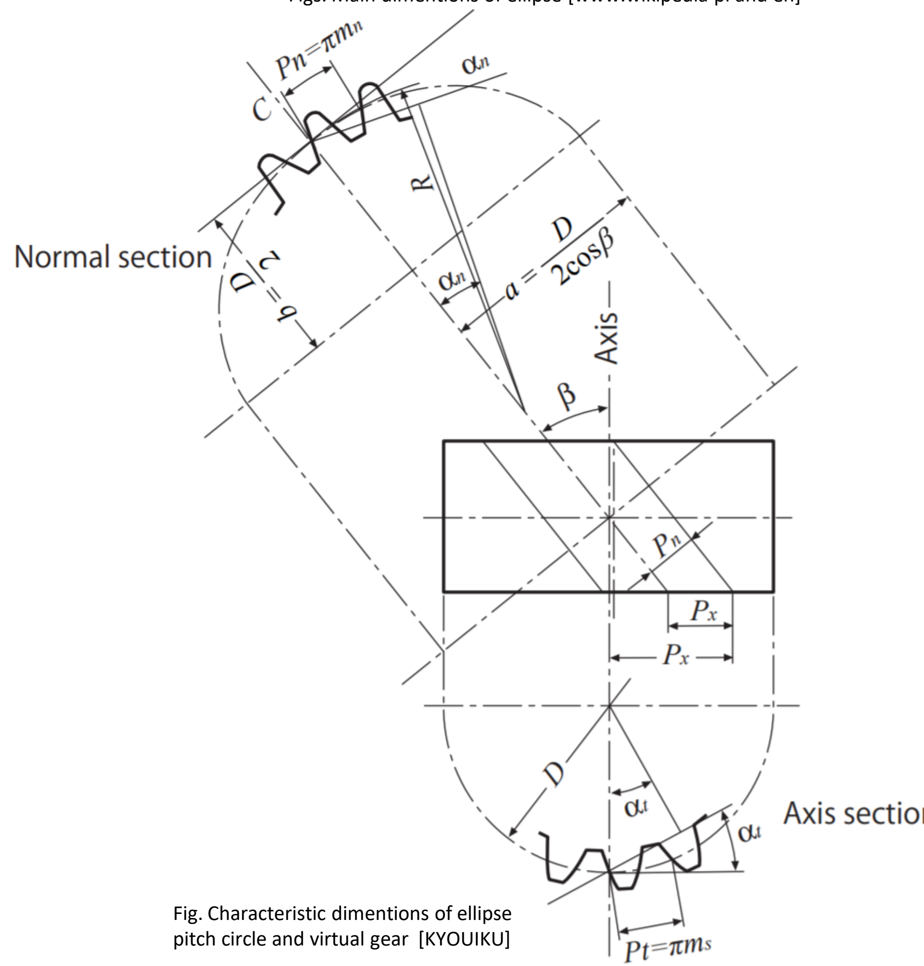


Fig. Characteristic dimentions of ellipse pitch circle and virtual gear [KYOUIKU]

Helical gear terminology (involute profile)

Equivalent (virtual) spur gear and equivalent (virtual) number of teeth

Equivalent number of teeth is a number of teeth on equivalent spur gear.

Determination of *equivalent number of teeth* z_n

Using general formula on pitch diameter it can be write

$$d_n = z_n m_n$$

and having defined pitch diameter of equivalent spur gear

$$d_n = \frac{d}{\cos^2 \beta}$$

these two equations can be compared

$$z_n m_n = \frac{d}{\cos^2 \beta}$$

and remembering that $d = \frac{z m_n}{\cos \beta}$

Finally it could be write

$$z_n = \frac{z}{\cos^3 \beta}$$

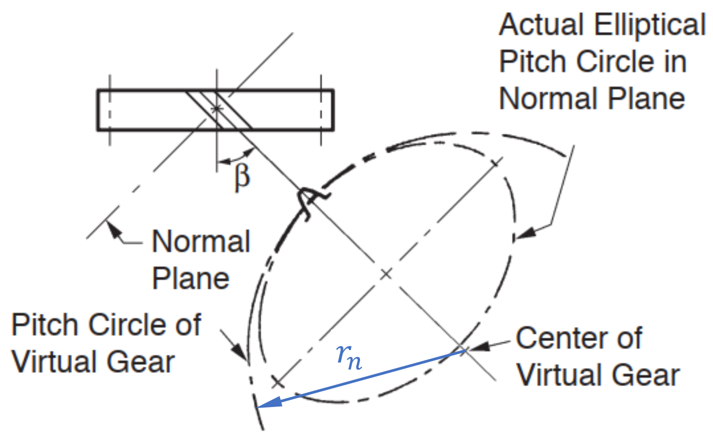


Fig. Ellipse pitch circle and pitch circle of virtual gear [KHK]

Gear ratio

Gear ratio i of fixed-axis gearing is relatively easy to determine using kinematic or geometric dependencies

$$i_{12} = \frac{\omega_1}{\omega_2} = \frac{z_2}{z_1}$$

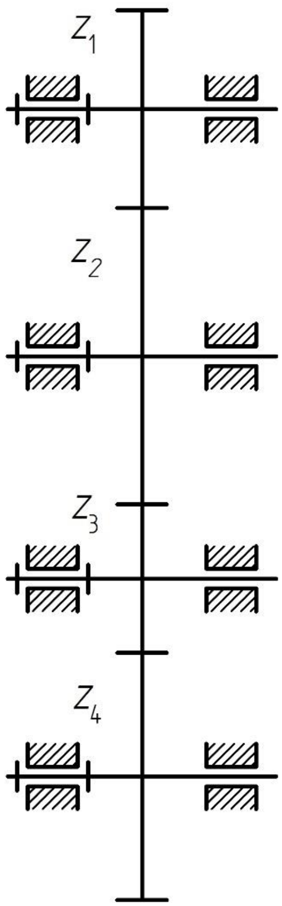
where:
ω_1 is the angular velocity of the driving gear (primary),
ω_2 is the angular velocity of the driven gear (secondary),
z_1 is the number of teeth of the driving gear,
z_2 is the number of teeth of the driven gear,

Rys. [http://www.paginasamarillas.com.gt
/empresas/cedema-sa/guatemala-15796039]

Gear ratio

Fixed-axis gears can come with two types of gearing: simple and complex. A simple gear train is one in which each shaft carries only one gear. A compound train is one in which at least one shaft carries more than one gear.
The gear ratio of a simple gear trains depends on parameters of the first and last gears, rather than on the number of teeth on the idler gears. The gear ratio of the gear train shown in Fig. is:

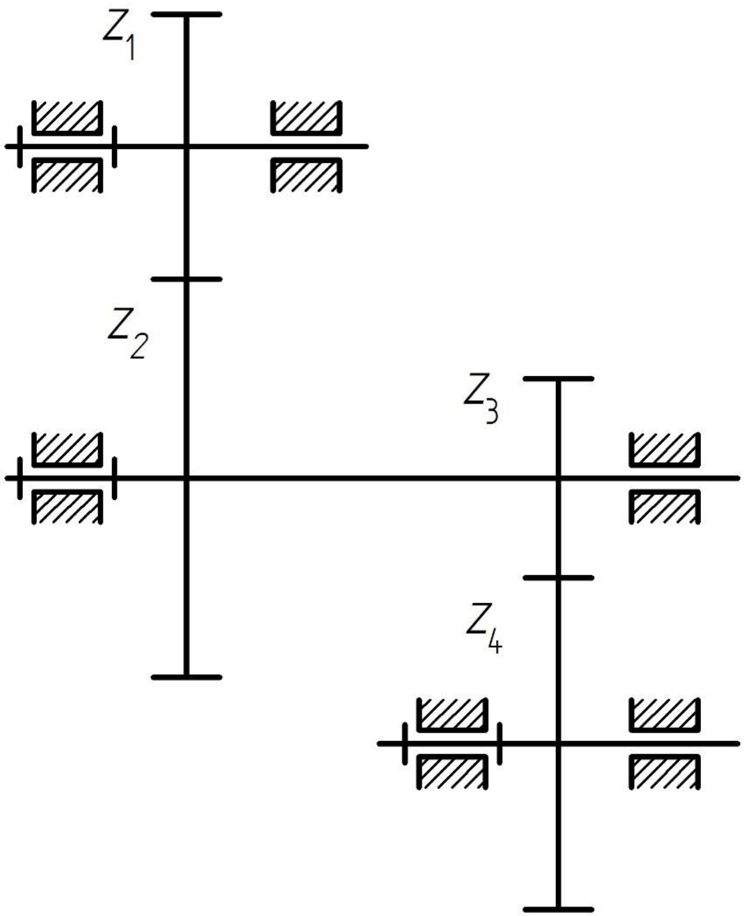
$$i_{14} = i_{12} \cdot i_{23} \cdot i_{34} = \frac{z_2}{z_1}\frac{z_3}{z_2}\frac{z_4}{z_3} = \frac{z_4}{z_1}$$



Gear ratio

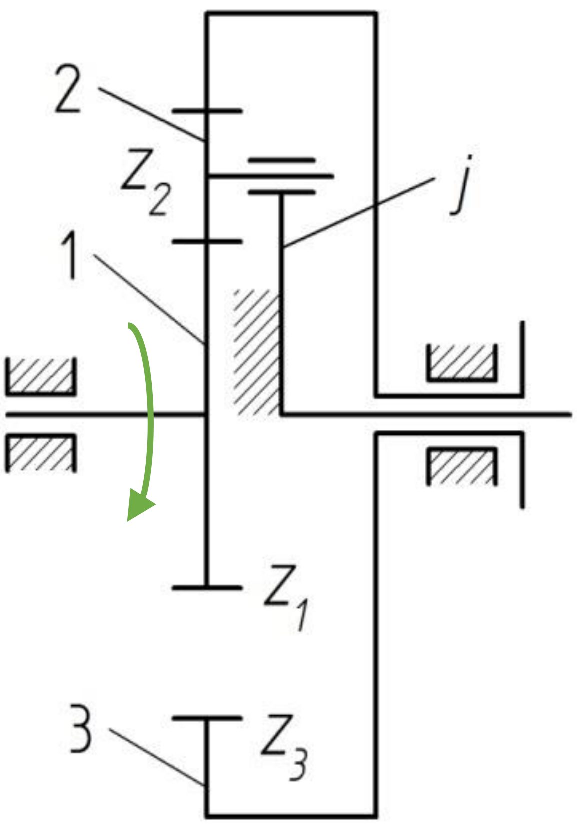
In compound gearing systems, the total gear ratio is the product of gear ratios of individual stages. The total gear ratio of the gear shown in Fig. is:

$$i_{14} = i_{12} \cdot i_{34} = \frac{z_2}{z_1}\frac{z_4}{z_3} = \frac{z_2 z_4}{z_3 z_1}$$



Gear ratio

Contrary to fixed-axis gearing where the gears only rotate, planetary gears have planet gears that perform plane motion. Planet gears can rotate around their own axis and, additionally, their axes can perform rotational motion, too. This makes determination of their gear ratio difficult. One of the methods for gear ratio determination is the Willis method. This method consists in changing the reference system from the housing to the planet carrier. This is done by assigning the angular velocity of the planet carrier yet in opposite direction to all parts. The Willis formula for determination of the gear ratio of planetary gears (Fig.) is:



$$i_{13}^{j} = \frac{\omega_1 - \omega_j}{\omega_3 - \omega_j}$$

i_{13}^{j} denotes the gear ratio between the driving gear 1 and the driven gear 3 with fixed planet carrier

Gear ratio

When planet carrier is not stationary, the above formula is rewritten such that the planet carrier becomes fixed. This can easily be done by exchanging the gear ratio subscripts and superscript in accordance with the two dependences:

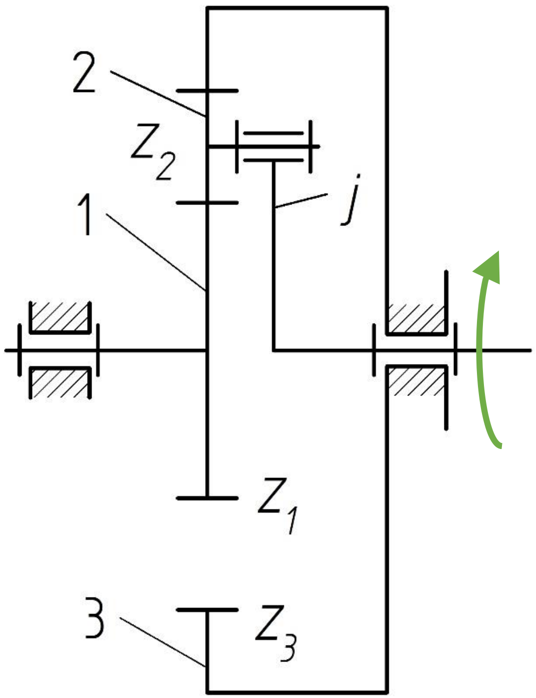
$$i_{j1}^{3} = \frac{1}{i_{1j}^{3}}$$

$$i_{1j}^{3} = 1 - i_{13}^{j}$$

In the case of planetary gears sign of gear ratio must be included. If the driving and driven gears rotate in the same direction, the gear ratio sign is positive. The sign is negative if the gears rotate in opposite directions.

Problem
$i_{j1}^{3} = ?$

$$i_{j1}^{3} = \frac{1}{i_{1j}^{3}} = \frac{1}{1 - i_{13}^{j}} = \frac{1}{1 - \left(-\frac{z_2}{z_1}\frac{z_3}{z_2}\right)} = \frac{1}{1 + \frac{z_3}{z_1}}$$



Gear forces

Information about forces acting on teeth that are in contact is vital to determine load of teeth, shafts, bearings, housing etc. Main force is force normal to teeth surface. The second is frictional force that is perpendicullar to normal. Value of frictional force is usually very low in compare to normal force and is neglected during determination of components.

Note if efficiency of gears is low friction force should be take into account.

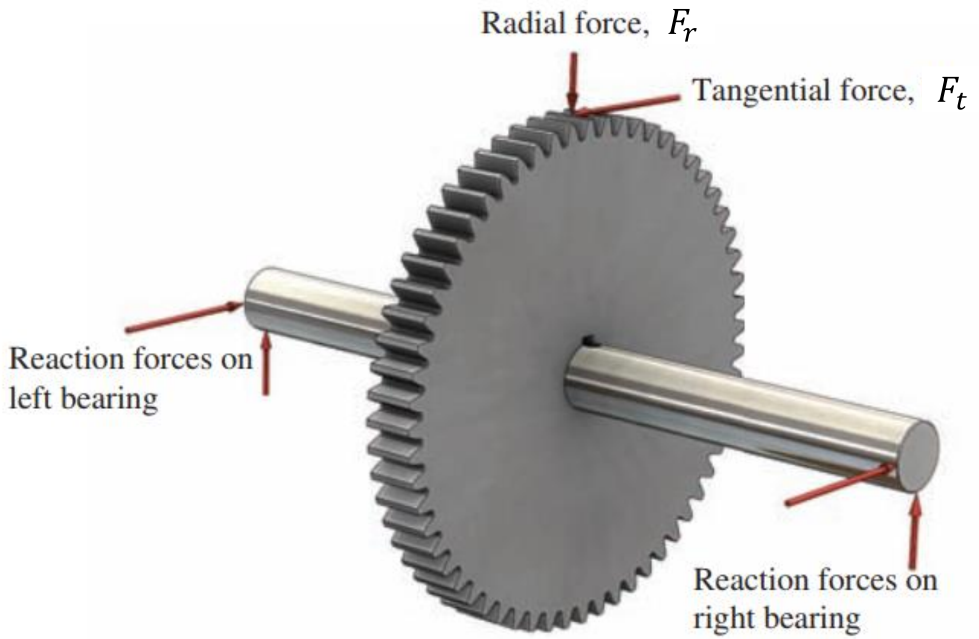


Fig. Load forces and reaction forces [Mott 2018]

Gear forces – spur gears

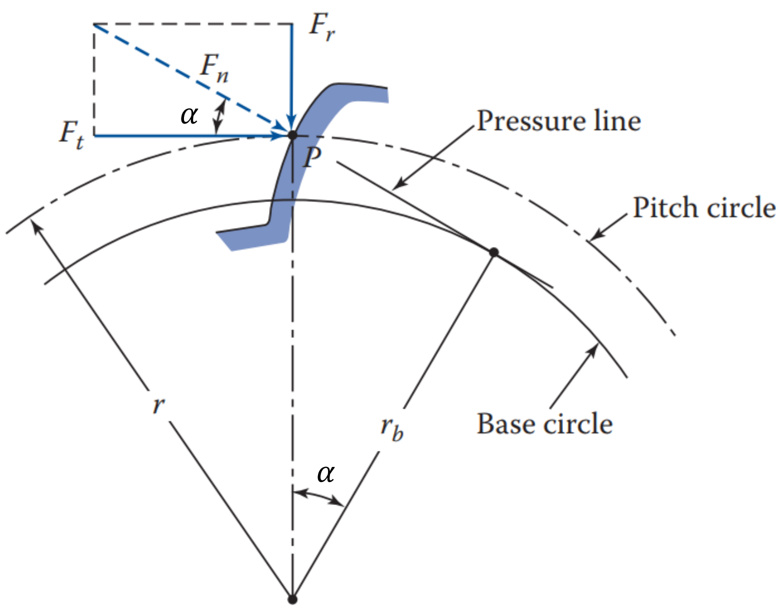


Fig. Components of normal force F_n in pitch point P: radial force F_r and tangential force F_t [Ugural 2015]

Torque transmitted by gears

$$M_t = \frac{P\ [W]}{\omega\ [rad/s]}\,[Nm] = 9554\,\frac{P\ [kW]}{n\ [rpm]}\ [Nm]$$

M_t - transmitted torque
P – transmitted power
ω – angular velocity
n – revolutions per minute

Normal force is divided into two components: radial and tangential. Only tangential component is useful

Relation between torque and tangenatial force (transmitted load)

$$F_t = \frac{M_t}{r}$$ r – radius of pitch circle

Normal force

$$F_n = \frac{F_t}{\cos\alpha}$$ α – pressure angle

Radial force

$$F_r = F_t \tan\alpha$$

For gears with shifted profiles

Tangenatial force

$$F_t = \frac{M_t}{r_w}$$ r_w – radius of working pitch circle

Normal force

$$F_n = \frac{F_t}{\cos\alpha_w}$$ α_w - working pressure angle

Radial force

$$F_r = F_t \tan\alpha_w$$

Gear forces – spur gears

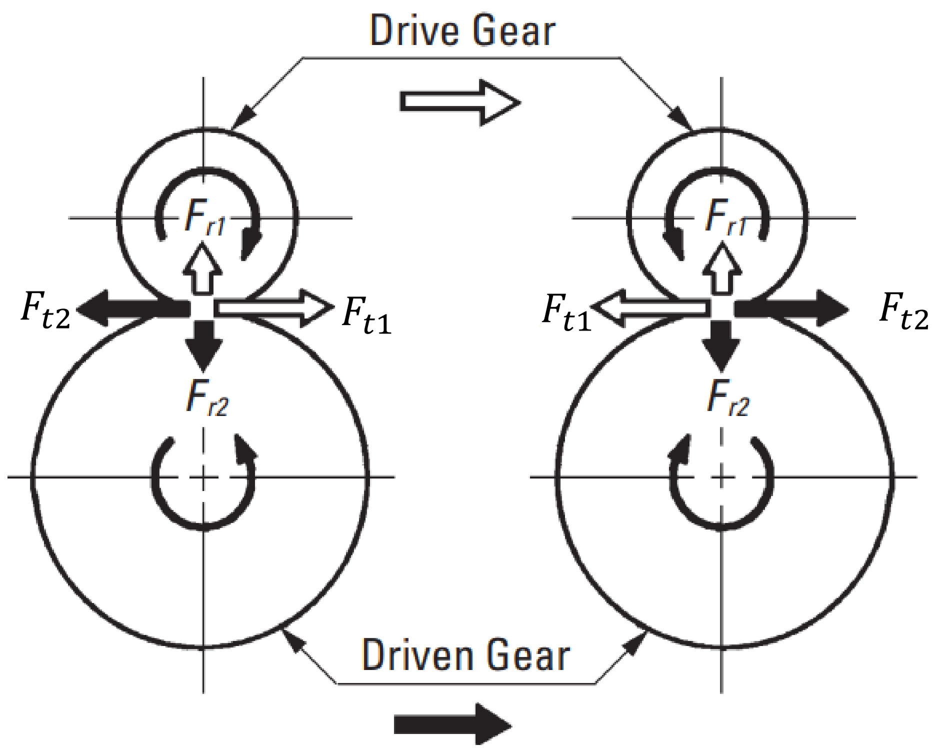


Fig. Reaction forces acting on spur gears in mesh [QTC METRIC GEARS]

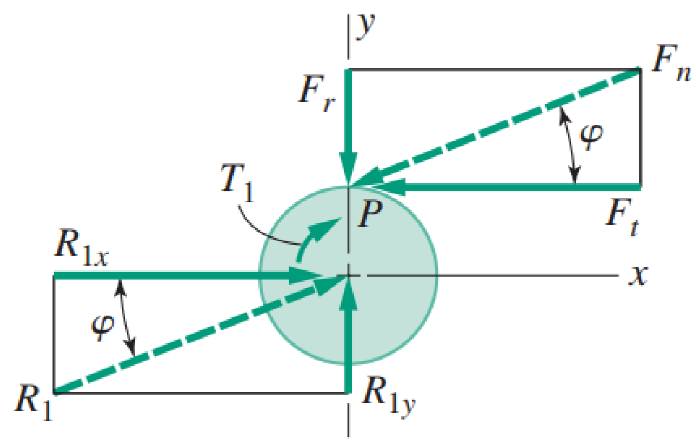


Fig. Reaction forces of shaft [Collins 2010]

Gear forces – helical gears

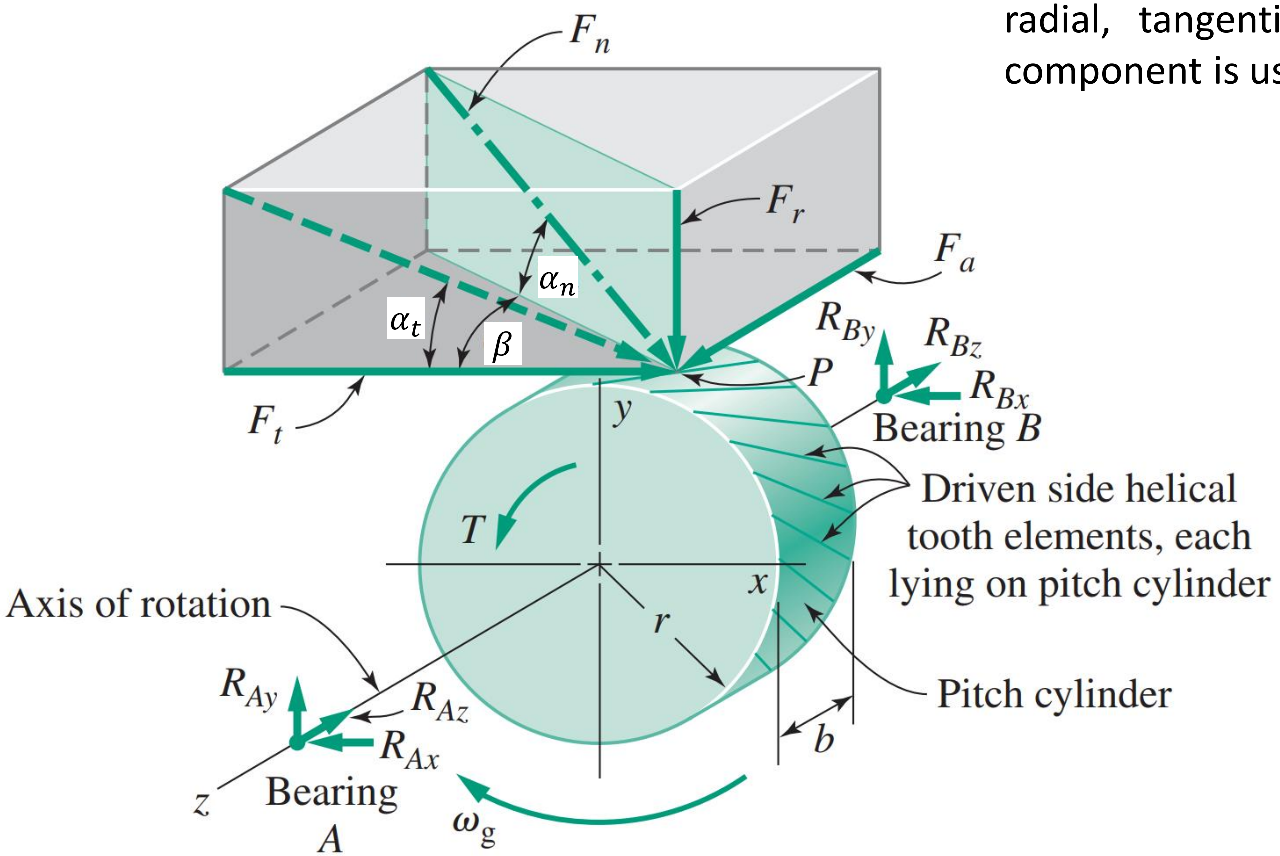


Fig. Distribution of forces in helical gear and bearing reactions [Collins 2010]

Normal force is divided into three components: radial, tangential and axial. Only tangential component is useful.

Radial force

$$F_r = F_t \tan\alpha_t = F_t \frac{\tan\alpha_n}{\cos\beta}$$

β – helix angle
α_t – transverse pressure angle
α_n – normal pressure angle

Axial force (thrust component)

$$F_a = F_t \tan\beta$$

Normal force

$$F_n = \frac{F_t}{\cos\beta\cos\alpha_n}$$

Tangential component is usually calculated from information about torque

Gear forces – helical gears

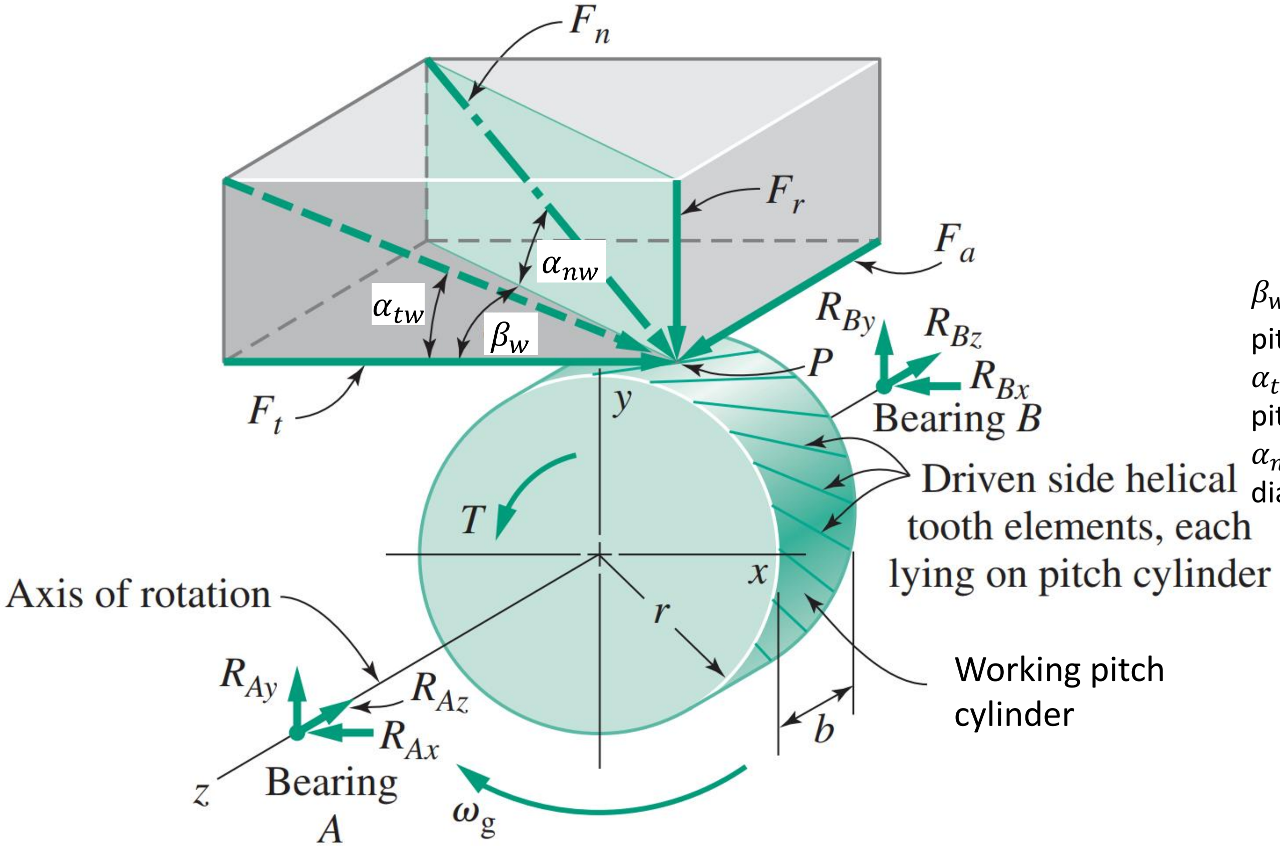


Fig. Distribution of forces in helical gear and bearing reactions [Collins 2010]

For gears with shifted profiles

Radial force

$$F_r = F_t \tan \alpha_{tw} = F_t \frac{\tan \alpha_{nw}}{\cos \beta_w}$$

$\beta_w = \tan^{-1} \frac{d_w \tan \beta}{d}$ helix angle on working pitch diameter

α_{tw} – transverse pressure angle on working pitch diameter

α_{nw} – normal pressure angle on working pitch diameter

Axial force (thrust component)

$$F_a = F_t \tan \beta_w$$

Normal force

$$F_n = \frac{F_t}{\cos \beta_w \cos \alpha_{nw}}$$

Gear forces – helical gears

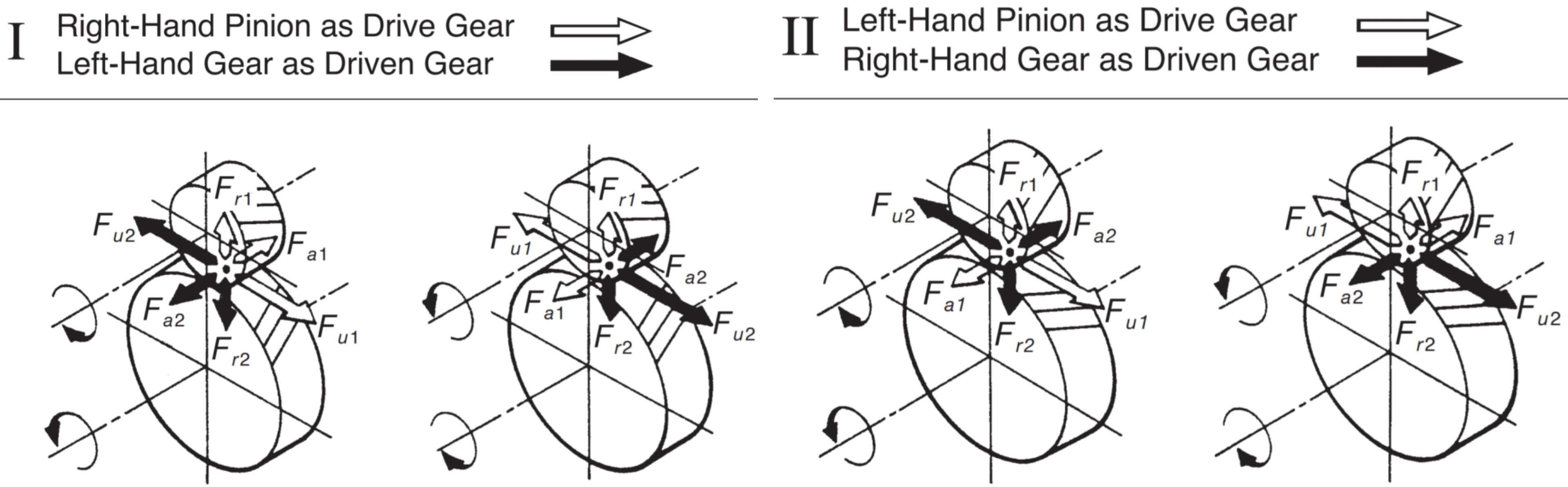


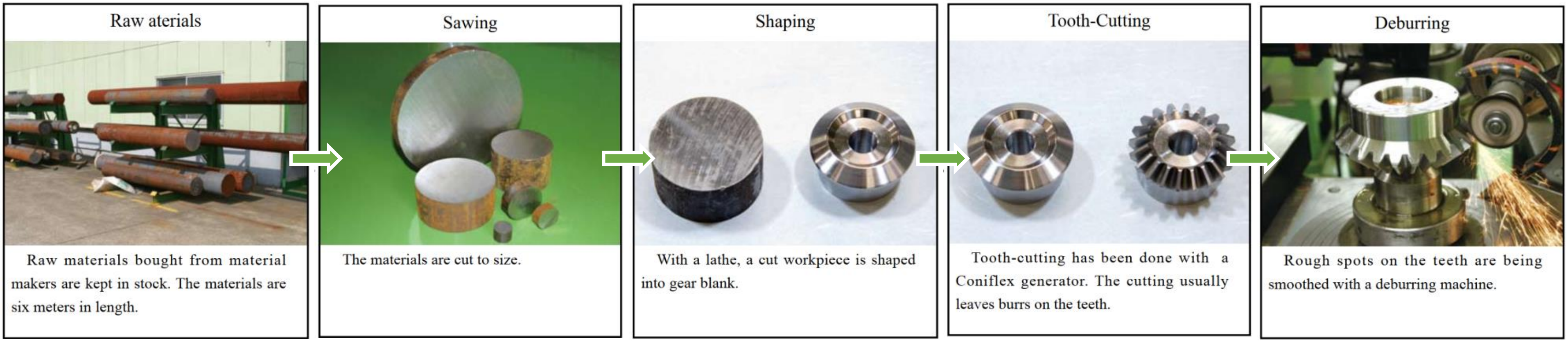
Fig. Reaction forces acting on helical gears in mesh [QTC METRIC GEARS]

Gear manufacturing

Gears are made from one piece of material or are composed of several parts for large ones.

Technology that is applied to produce gears depend on: material, size, required strength and accuracy, cost, production volume, the company's production capacity.

Figures [www.indiamart.com]



Figures [KHK]

Gear manufacturing

Gears often are made by: machining (mainly metal gears), casting (mainly plastic and some metals), forming and powder matallurgy process. Machining can be applied after other method.

There will be presented ***only*** a short description of ***machining methods*** most common for metal gears.

Geometry of gear teeth was associated with method of manufacturing. Expressive example are spiral bevel gears. Currently mainly to multi-axis CNC machines geometry of teeth can be as was design.

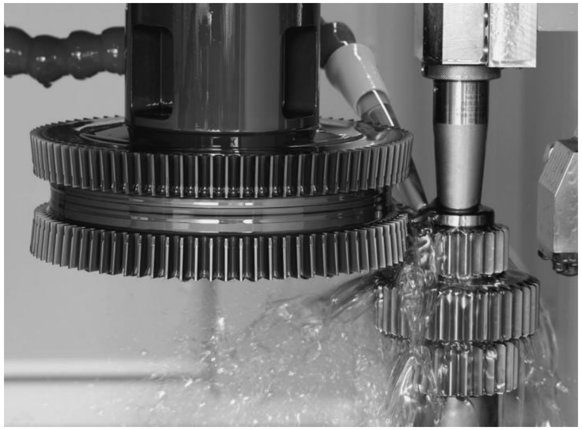


Fig. Machining [Radzevich 2017]

Fig. Powder metal gear
[https://www.geartechnology.com/articles/0917/What's_New_and_Noteworthy_in_Powder_Metal/]

Fig. Cold formed gear (cold form process was final)
[https://www.aikoku.co.jp/en/cf/products/04.html]

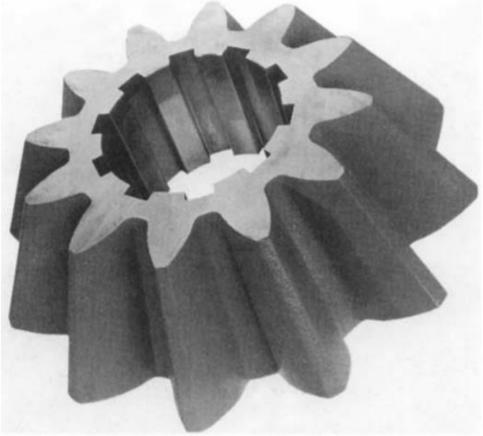


Fig. „*Cast tooth pinion gear for an electric mining shovel. Weight: 212 kg*"
[https://www.asminternational.org/documents/10192/3478857/Chapter_6_WEB_2.pdf/5423f450-7277-4fc4-b332-9518e95f3d74]

Gear manufacturing

Methods of cutting gears may be divided into three types:

- generating,
- form,
- universal.

After cutting teeth they can be finished by the process like: grinding, lapping, shaving or honing (not all methods can be use to all types of gears)

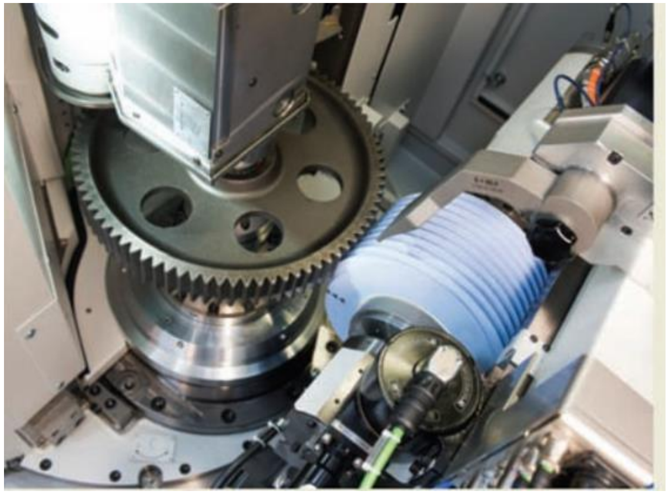


Fig. Grinding [Mott 2018]

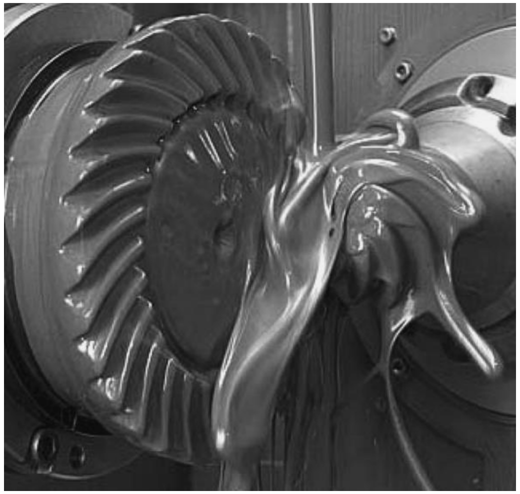


Fig. Lapping [Radzevich 2017]

Fig. Shaving [Radzevich 2017]

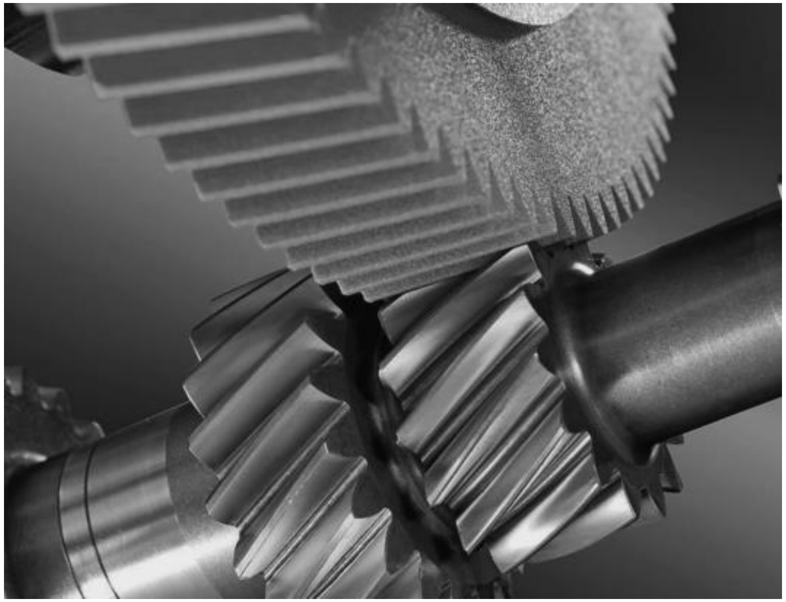


Fig. Honing [Radzevich 2017]

Gear manufacturing

Methods of cutting gears may be divided into three types:

- generating,
- form,
- universal.

In generating method tool has different shape than tooth profile. Shape of tooth is generating during relative motion of gear blank and tool. Tool is special design to cut teeth. This method is fast, precise and suitable for large production volume. The drawback is cost of tools and machines.

Fig. Rack cutter [Radzevich 2017]

Fig. Gear shaper cutter - spur geas [Radzevich 2017]

Fig. Gear shaper cutter - hellical geas [Radzevich 2017]

Fig. Hob cutter [Radzevich 2017]

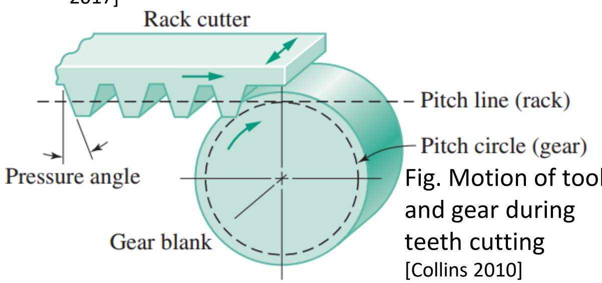


Fig. Motion of tool and gear during teeth cutting [Collins 2010]

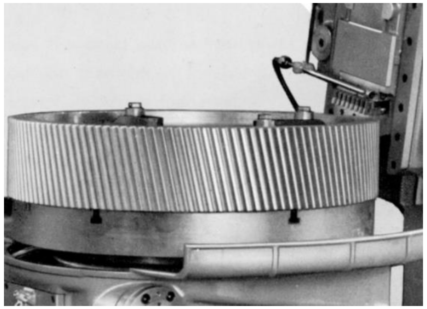


Fig. Cutting teeth by rack cutter – Maag method [http://www.lathes.co.uk/maag/]

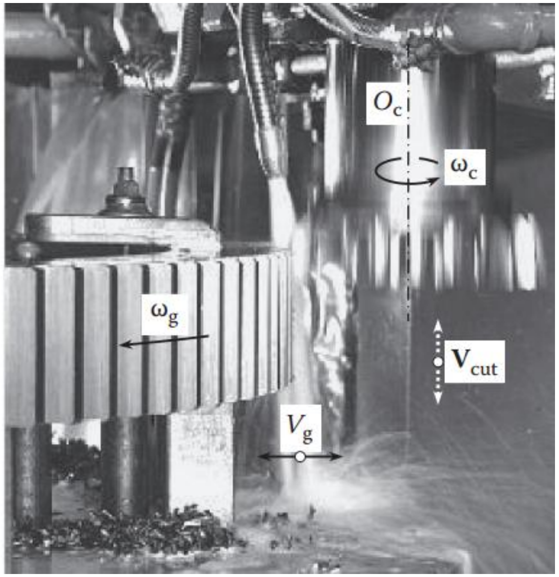


Fig. Motion of tool and gear during cutting spur teeth - Fellows method [Radzevich 2017]

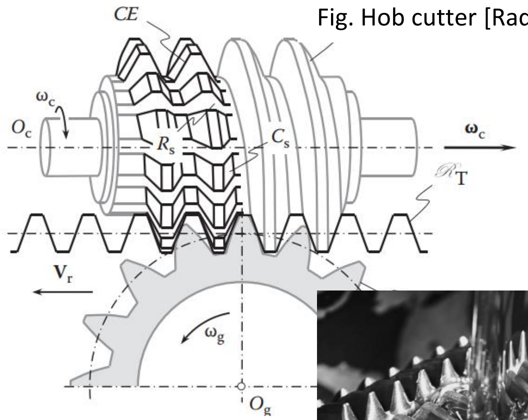


Fig. Motion of tool and gear during teeth cutting [Radzevich 2017]

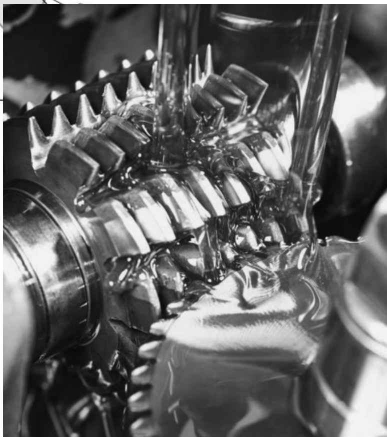


Fig. Cutting teeth by hub cutter [Radzevich 2017]

Gear manufacturing

In generating method tool has different shape than tooth profile. Shape of tooth is generating during relative motion of gear blank and tool. Tool is special design to cut teeth. This method is fast, precise and suitable for large production volume. The drawback is cost of tools and machines.

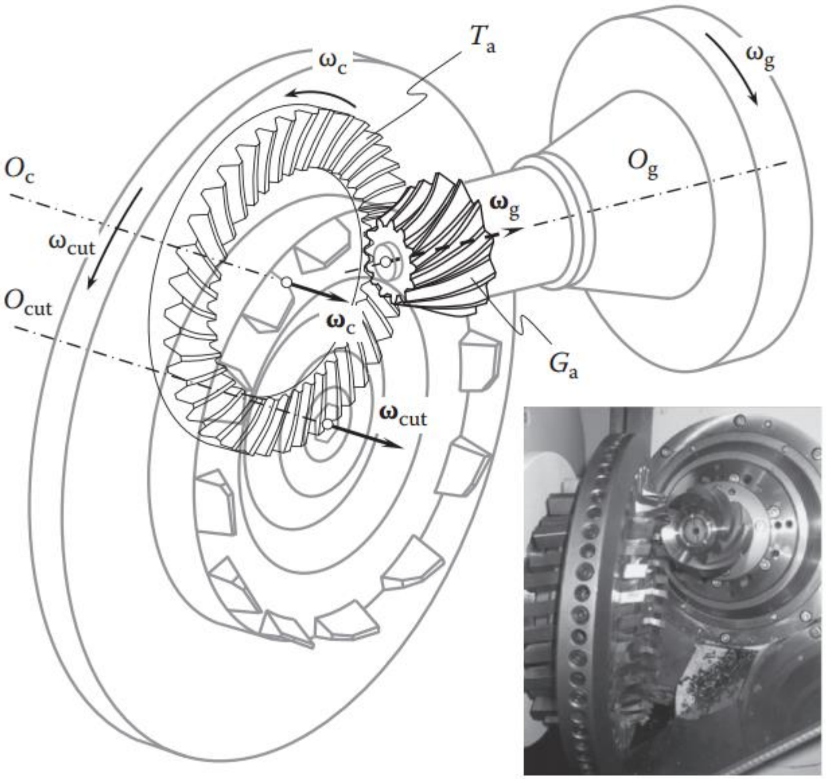


Fig. Motion of tool and gear during teeth cutting [Radzevich 2017]

Fig. Cutter heads for spiral bevel gears [Radzevich 2017]

Gear manufacturing

Methods of cutting gears may be divided into three types:

- generating,
- form,
- universal.

In form method tool has identical shape as tooth space. Tool is specialy design to cut teeth but the process is usually done on universal machines. This method is suitable for small production volume when high precision is not required. Shape of tooth space is different for different numer of teeth but tools are ususally design to have correct shape for the possible smallest numer of teeth. Cutting larger number of teeth introduced profile error.

Fig. End mill gear cutters [http://www.supercapitaltools.com/products1.htm]

Fig. Disc mill gear cutters [https://www.vargus.com/Vargus/userdata/SendFile.asp?DBID=1&LNGID=1&GID=218]

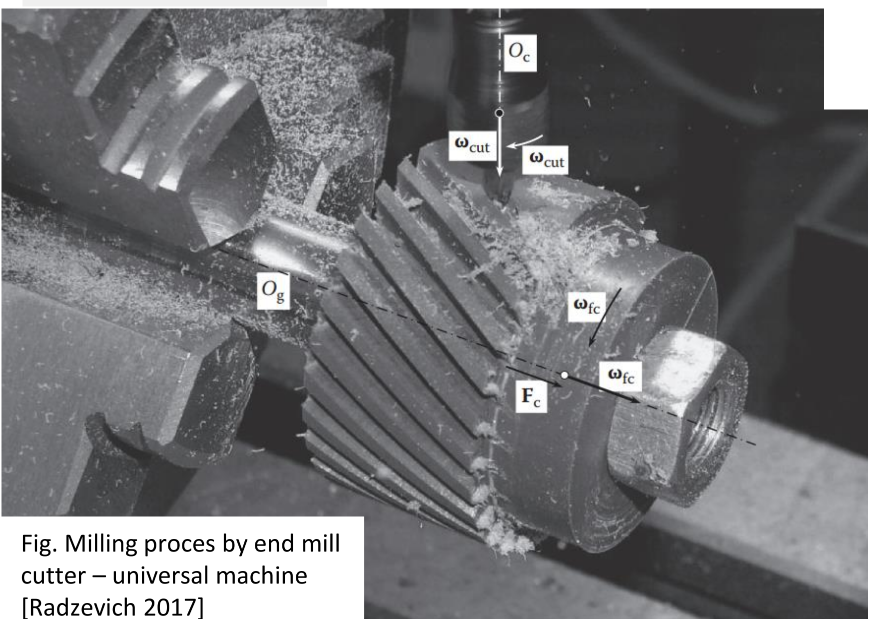


Fig. Milling proces by end mill cutter – universal machine [Radzevich 2017]

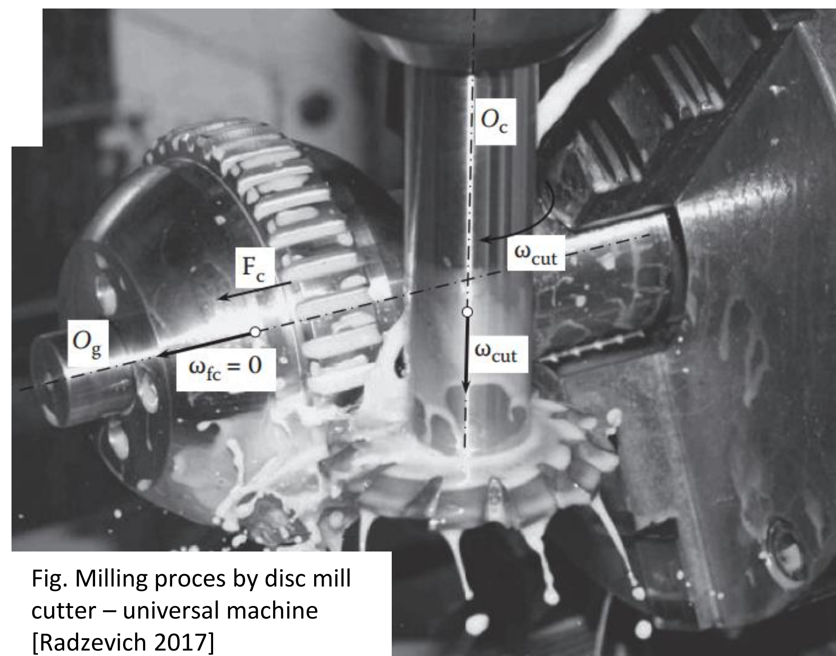


Fig. Milling proces by disc mill cutter – universal machine [Radzevich 2017]

Gear manufacturing

Methods of cutting gears may be divided into three types:

- generating,
- form,
- universal.

In form method tool has identical shape as tooth space. Tool is specialy design to cut teeth but the process is usually done on universal machines. This method is suitable for small production volume when high precision is not required. Shape of tooth space is different for different numer of teeth but tools are ususally design to have correct shape for the possible smallest numer of teeth. Cutting larger number of teeth introduced profile error.

Fig. Broach [Radzevich 2017]

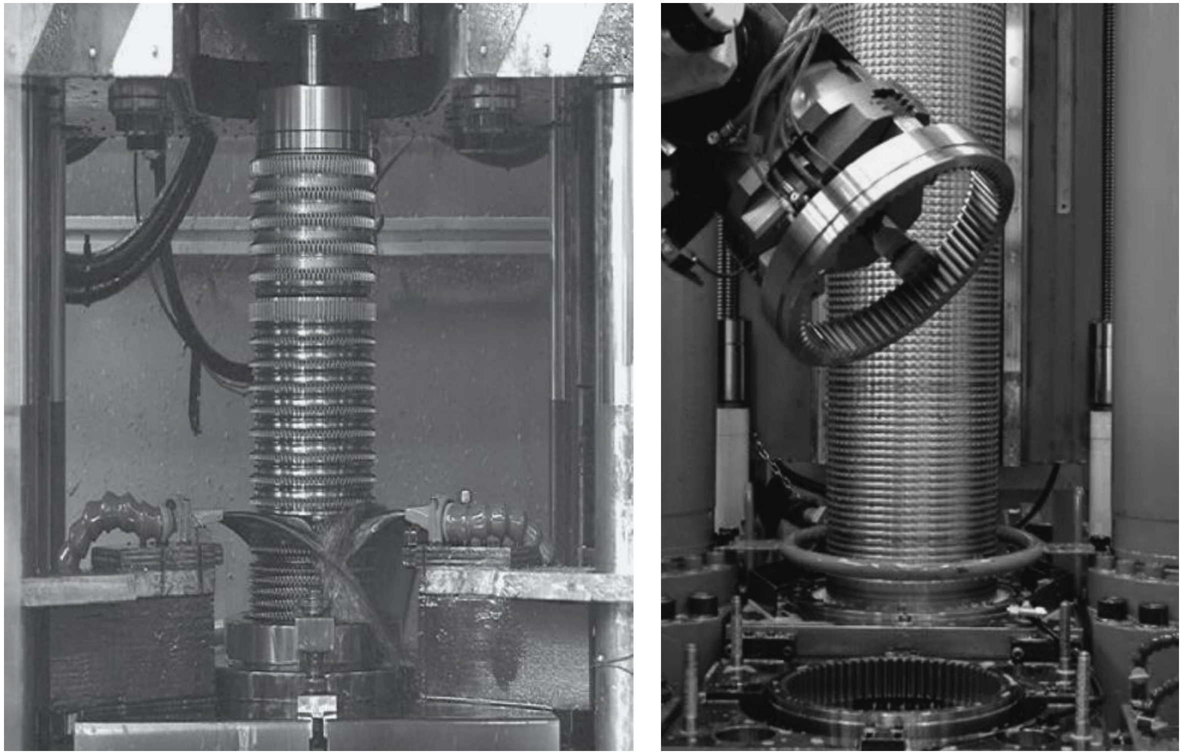


Fig. Broaching process [Radzevich 2017]

Gear manufacturing

Methods of cutting gears may be divided into three types:

- generating,
- form,
- universal.

Universal method is the newest and have the beginning with developmet of multi-axis CNC machines that are capable to create complicated surface. Tool and machine are usually universal. This method is suitable for small production volume and manufacturing gears with complicated teeth shape like in spiral bevel gears.

Fig. Milling proces by end mill cutter – 5 axis CNC machine [Radzevich 2017]

Fig. Milling proces by disc mill cutter – 5 axis CNC machine [https://www.geartechnology.com/articles/0317/5-Axis_Gear_Manufacturing_Gets_Practical/]

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