Lublin University of Technology

Theory of Machine

Lecture

Kinematic analysis

Lublin 2020

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General information

Kinematics is the study of motion without taking into account its causes. The description of the movement requires two basic units: length in meters and time in seconds. The remaining units are derivatives. Motion is a relative concept and its description depends on the adopted reference system.

In the analysis task it is assumed that the movement of the driving member is known, and the parameters of the movement of the remaining members of the mechanism relative to the base (frame) are searched. The purpose of the kinematic analysis may be to determine:

- position of links or paths of points,
- velocity,
- acceleration.

General information

Rigid body motion types:

1. Plane motion:
- translation - rectilinear and curvilinear,
- rotation.

2. Spatial motion:
- rotation about fixed point,
- helical,
- general.

Plane motion of a rigid body occurs when all points of the body move in planes parallel to some stationary plane.
By extending this definition to all movable links of a mechanism, a group is distinguished called plane mechanisms.
Most of the mechanisms used in practice belong to this group.

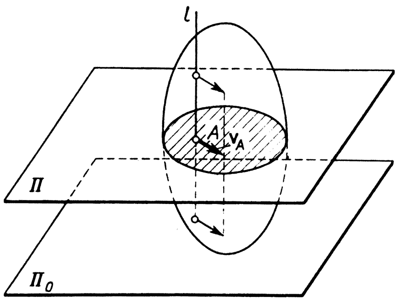


Fig. [Leyko 2012]

General information

1. Translational motion

2. Rotational motion

Plane motion

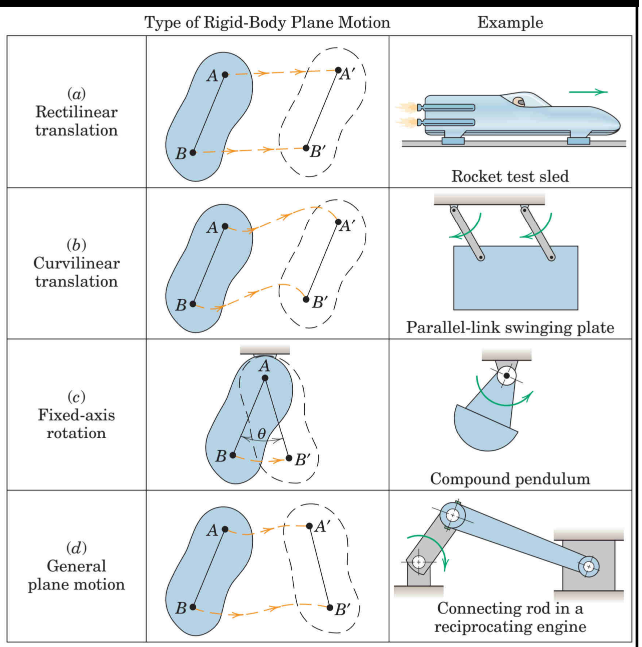


Fig. [Meriam 2012]

General information

Spatial motion

General motion

Fig. [http://www.4wdmechanix.com/Video-Youth-Wins-Ride-in-4WD-Desert-Race-Car%21?r=1]

Rotation about fixed point

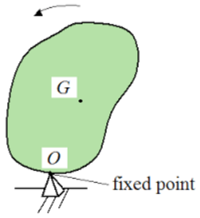


Fig. [http://www.real-world-physics-problems.com/kinetic-energy.html]

Helical motion

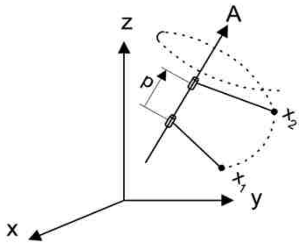
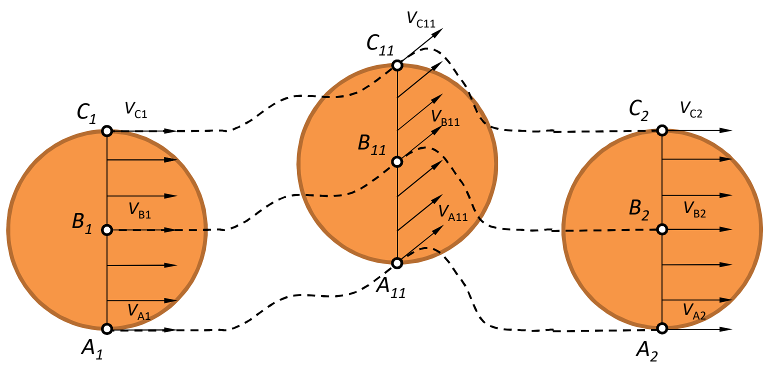


Fig. [http://biomechanical.asmedigitalcollection.asme.org/article.aspx?articleid=1841517]

Fig. [http://kmoddl.library.cornell.edu/model.php?m=1]

Translational motion

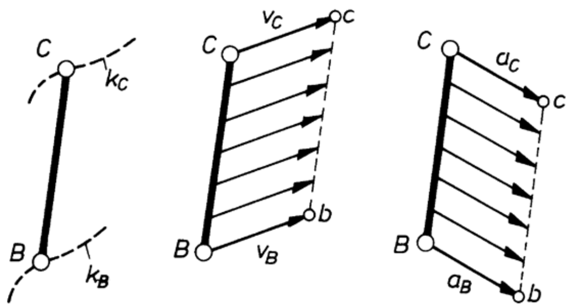
"*A link is in **translational motion** when any section associated with this link maintains a parallel position in all phases of motion*" [Miller 1996].
Therefore, in this motion, the paths of all points of the link are identical, as are the velocities and accelerations.
In rectilinear motion, the paths are a straight line and in a curvilinear motion, they are a curve.



Translational motion

$v = ds/dt,$ Velocity [m/s]

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2 s}{dt^2}$$ Acceleration [m/s²]

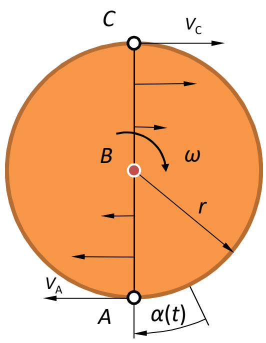


$$v_B = v_C = v_i, \quad \omega = 0,$$
$$a_B = a_C = a_i, \quad \varepsilon = 0,$$

Fig. [Miller 1996]

Rotational motion

Rotation occurs when the paths of all points on the body draw circles whose centers lie on a common straight line known as the axis of rotation. This means that each point on the body rotates the same angle.



$$\omega = \frac{d\alpha}{dt} = \dot{\alpha}$$ Angular velocity [rad/s]

$$v_A = v_C = \omega \cdot r,$$ Linear velocity [m/s]

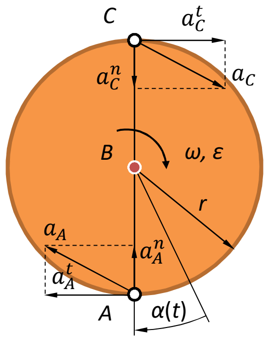
$$v_B = 0.$$

Fig. [http://energyeducation.ca/encyclopedia/Gear]

$$\omega = \frac{\pi n}{30}\,[\text{rad/s}]$$ n [rpm]

Rotational motion

$$\varepsilon = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\alpha}{dt}\right) = \frac{d^2\alpha}{dt^2} = \ddot{\alpha}$$ Angular acceleration [rad/s²]



$$a_n = \omega^2 r = \frac{v^2}{r}$$ Normal (radial, centripetal) acceleration

$$a_t = \varepsilon r$$ Tangential acceleration

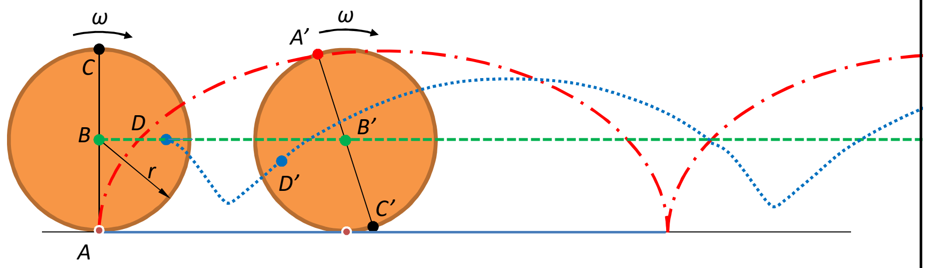
$$a = \sqrt{a_n^2 + a_t^2} = r\sqrt{\omega^4 + \varepsilon^2}$$

$$a_A = a_c$$

$$a_B = 0$$

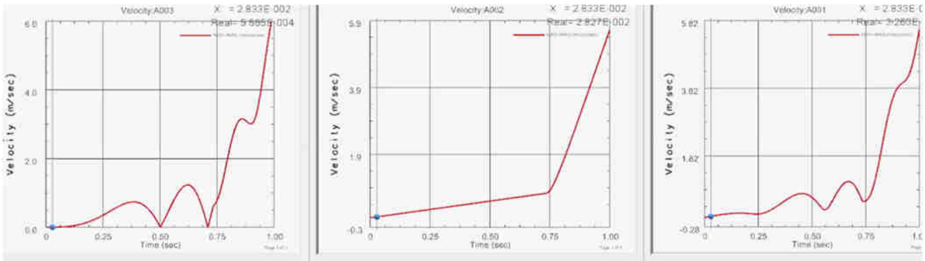
Plane motion

To remind, plane motion of a rigid body occurs when all points of the body move in planes parallel to some stationary plane. Translational and rotary motion are special cases of plane motion.



Plane motion

The paths of the points, velocity and acceleration values are generally different from each other



Instantaneous centre (on the begining of simulation) | Center of the circle | Blue point

Plane motion – method of analysis

1. Based on the equation of motion and its form after differentiation, positions, velocities and accelerations are determined.

In order to facilitate the analysis of planar motion it can be treated as:

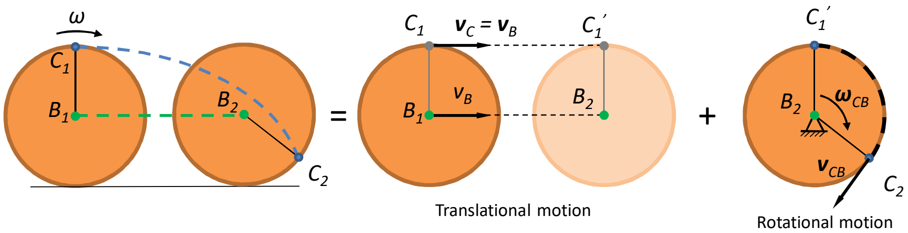
1. Composed of translational and rotational motion.
2. Rotation about the instantaneous center.
3. The third possibility to calculate the velocity in this motion is to use the relationship between the velocities of the points of a rigid body (velocity projection method).

Plane motion as a motion composed of translational and rotational motion

This method makes it possible to determine the velocity of the point of a rigid body by graphical approach if the following is known:

- velocity of one point (value, direction),
- direction of the searched velocity.

The displacement of a link, in an infinitely short period of time, is treated as composed of two motions - translational and rotational. In the first step, the wheel moves at speed $\boldsymbol{v}_B$ from point B_1 to B_2, and then rotates in relation to this point B_2 until point C_1' reaches position C_2. The points on the link can be chosen freely.



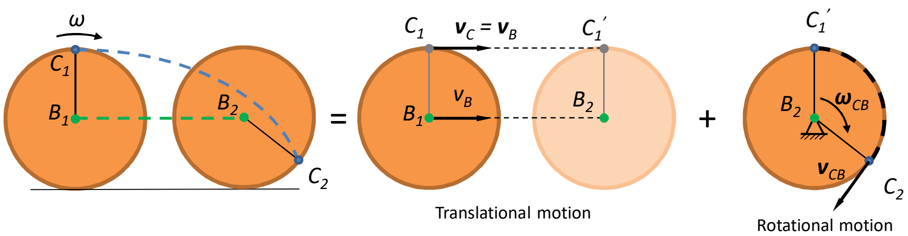
Plane motion as a motion composed of translational and rotational motion

The velocity of a point C is equal to the sum of the velocity of point B in translational motion and the velocity of point C relative to the point B in rotational motion:

$$\vec{v}_C = \vec{v}_B + \vec{v}_{CB}$$

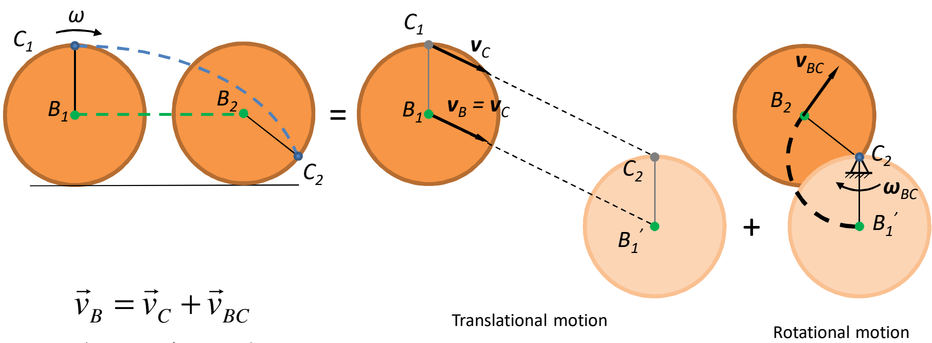
The relative velocity is always perpendicular to the straight line passing through the points under consideration and is equal to:

$$v_{CB} = \omega_{CB} r$$



Plane motion as a motion composed of translational and rotational motion

The same displacement can be considered as translational motion of the rigid body to change the position of the point from C_1 to C_2, and then rotating about that point so that point B_1' reaches position B_2.



$$\vec{v}_B = \vec{v}_C + \vec{v}_{BC}$$

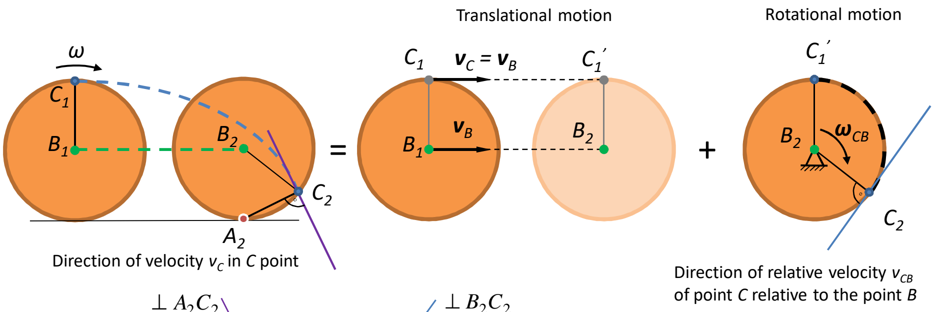
$$\vec{v}_{BC} = \vec{\omega}_{BC} \times \vec{r}$$

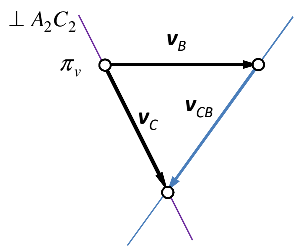
$$\vec{v}_{BC} = -\vec{v}_{CB}$$

Relative velocities $\boldsymbol{v}_{BC} = -\boldsymbol{v}_{CB}$ of the links's points have the same magnitude and opposite sens

Plane motion as a motion composed of translational and rotational motion

Having information about the velocity of one point and the direction of the unknown velocity and the relative speed, we can determine it graphically.





$$\underset{\perp A_2C_2}{\vec{v}_C} = \underset{\| B_1B_2}{\vec{v}_B} + \underset{\perp B_2C_2}{\vec{v}_{CB}}$$

Plane motion as a motion composed of translational and rotational motion

The acceleration of a point is determined analogously as the sum of the acceleration in translational motion and the relative acceleration of a point in rotational motion.

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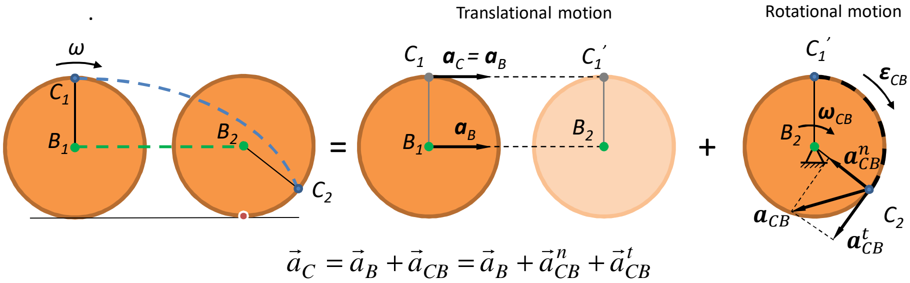


$$\vec{a}_C = \vec{a}_B + \vec{a}_{CB} = \vec{a}_B + \vec{a}_{CB}^n + \vec{a}_{CB}^t$$

The relative acceleration a_{CB} consists of normal and tangential acceleration because the link rotates:

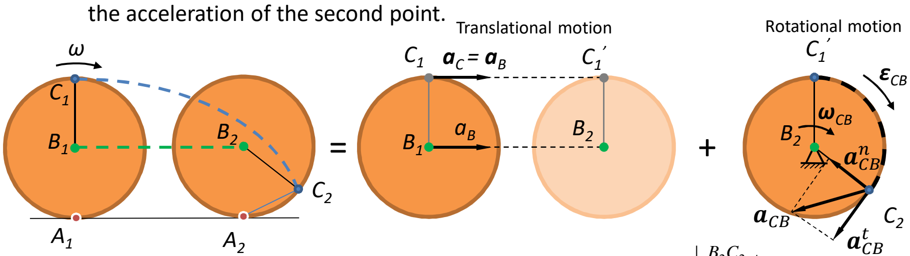
$$\vec{a}_{CB} = \vec{a}_{CB}^n + \vec{a}_{CB}^t$$

$$a_{CB}^n = \omega_{CB}{}^2 r = \frac{v_{CB}^2}{r}$$

$$\vec{a}_{CB}^t = \varepsilon_{CB} \times r$$

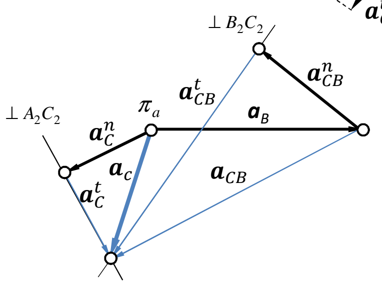
Plane motion as a motion composed of translational and rotational motion

Knowing the acceleration of one point of the rigid body and the relative normal acceleration, the direction of the tangential acceleration and the direction of the unknown acceleration, it is possible to graphically determine the acceleration of the second point.



$$\vec{a}_C = \vec{a}_B + \vec{a}_{CB} = \underset{\|B_1B_2}{\underline{\underline{\vec{a}_B}}} + \underset{\|B_2C_2}{\underline{\underline{\vec{a}_{CB}^n}}} + \underset{\perp B_2C_2}{\underline{\vec{a}_{CB}^t}}$$

$$\vec{a}_C = \underset{\|A_2C_2}{\underline{\underline{\vec{a}_C^n}}} + \underset{\perp A_2C_2}{\underline{\vec{a}_C^t}}$$

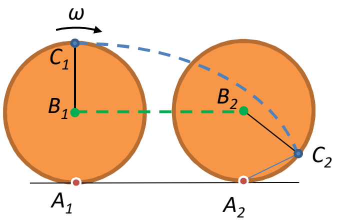


Plane motion as a motion composed of translational and rotational motion

If the acceleration of point B is known and the acceleration of point C is determined, the method is analogous and the following relationships apply:

$$\vec{a}_B = \vec{a}_C + \vec{a}_{BC}$$

$$\vec{a}_{CB} = -\vec{a}_{BC}$$



The given formulas are correct for any chosen points

Plane motion as an instantaneous rotational motion

In order to determine the velocity of any point of a rigid body, in practice, it is required to be known:

- velocity of one point,
- the location of the instantaneous center.

Any plane motion of the link can be represented as an infinitely short rotation about the instantaneous center. This center has a velocity of zero only at the considered moment. It's position is different in next moment.

Plane motion as an instantaneous rotational motion

Examples of determining the location of instantaneous centers are presented on below figures

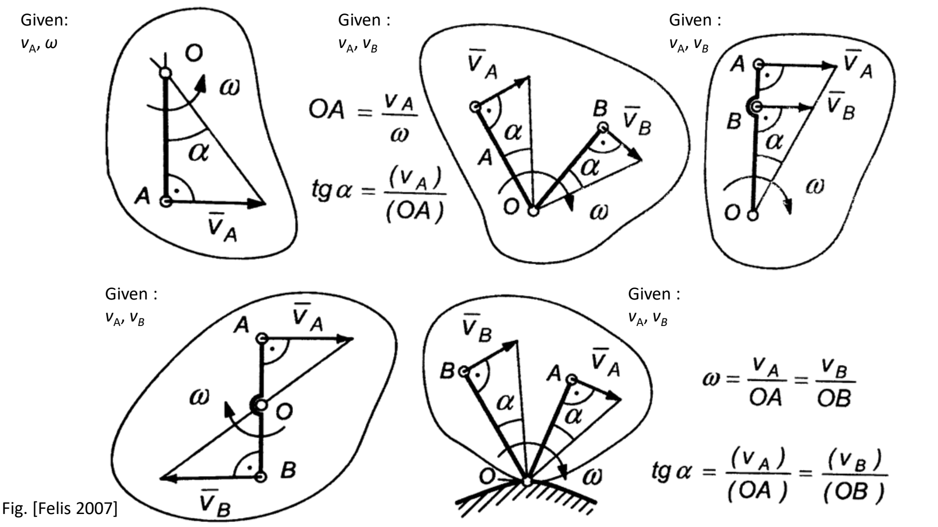
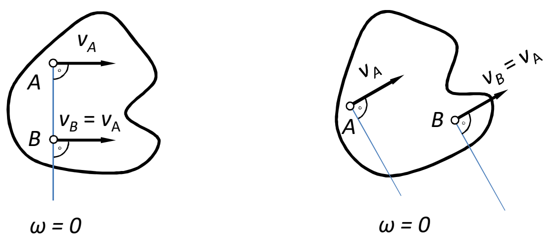


Fig. [Felis 2007]

Plane motion as an instantaneous rotational motion

If the link makes translation motion, for which the angular velocity is zero, the instantaneous centre do not exist.



Plane motion as an instantaneous rotational motion

In order to determine the acceleration of any point of a rigid body in practice, the knowledge is required about:

- the acceleration of two points.

Analogously to the instantaneous centre (of rotation) exist instantaneous centre of acceleration, i.e. point about which acceleration at given moment is equal zero.
It is not a frequently used method because of the required information of the acceleration of two points. The potential use may be in the case of links with more than two nodes.
The angle β that forms the acceleration vector with the straight line passing through the instantaneous center of acceleration S and the beginning of the acceleration vector is constant and does not depend on the point position. It is always an acute angle.

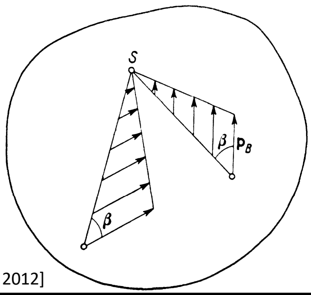


Fig. [Leyko 2012]

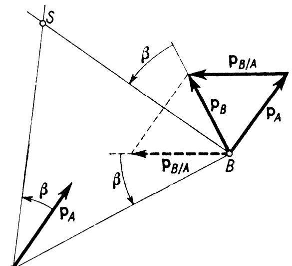


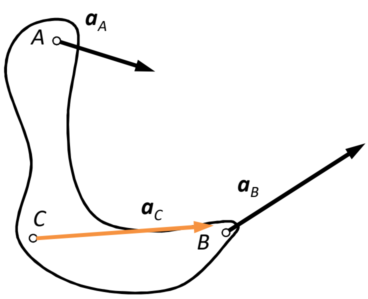
Fig. [Leyko 2012]

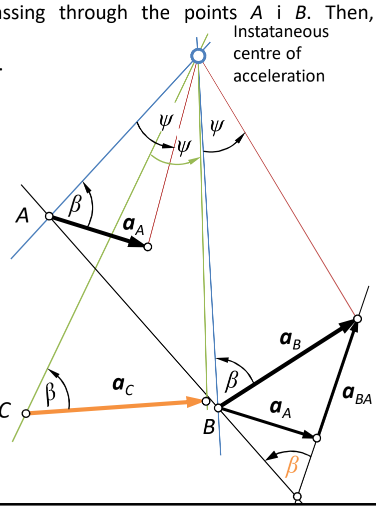
Plane motion as an instantaneous rotational motion

Example

Find the acceleration of point C of the rigid body using the method of instantaneous centre of acceleration. Given: acceleration of points A and B.
The solution uses the relationship $\vec{a}_B = \vec{a}_A + \vec{a}_{BA}$ to obtain the angle β, determined by the relative acceleration a_{BA} and the straight line passing through the points A i B. Then, remembering that the angle ψ is constant for all point of link, the acceleration of point C is determined.

More details can be find in Leyko 2012 and Młynarski 1992 (in polish).



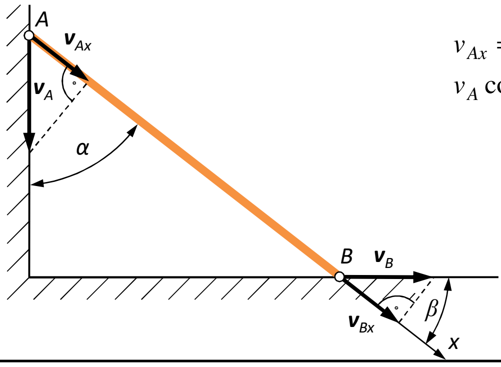


Plane motion - velocity projection method

This method makes it possible to determine the velocity of the point of a rigid body, if known:

- velocity of one point,
- direction of the searched velocity.

The relationship between the velocities of the points of a rigid body results from the constant distance between them.
This method is based on the relationship that projections of velocities of any points of a rigid body, lying on a common straight line, on this straight line, are equal to each other.



$$v_{Ax} = v_{Bx}$$
$$v_A \cos(\alpha) = v_B \cos(\beta)$$

Compound motion

There are mechanisms in engineering in which one link moves after another link that is also in motion. A good example is the movable guide and slider. Direct determination of the velocity and acceleration of the slider in relation to the base (stationary reference frame) is a difficult task. Therefore, the movement of the slider can be divided and considered in relation to the second movable reference frame associated with the guide. The following terminology is used to describe these motions:

- ***absolute motion*** - the motion of the element in relation to the stationary reference frame,
- ***relative motion*** - the motion of the element in relation to the movable reference frame,
- ***lifting motion*** - the motion of a movable reference frame in relation to the stationary reference frame.

In the example of the movable guide and the slider, the absolute motion is the motion of slider respect to the base, the relative motion is the motion of the slider respect to the guide, the lifting motion is the motion of the guide respect to the base.

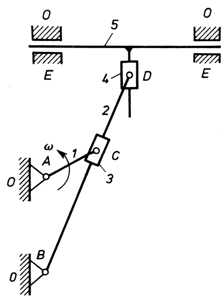


Fig. Mechanism of planing machine [Morecki 1987]

Compound motion

The formulas for calculating velocity and acceleration with the use of compound motion will be presented on the example of a slider and a moving guide.
The slider (absolute) velocity v_C at point *C* is equal to:

$$\vec{v}_C = \vec{v}_B + \vec{v}_{CB}$$

$\boldsymbol{v}_{CB}$ - it is the relative velocity (in relative motion) of point *C* with respect to *B* and it is always tangent to the guide (we imagine that the guide is stationary and slider is moving).

$\boldsymbol{v}_B$ – is the velocity of point *B*, i.e. lifting velocity. Point *B* belongs to guide and at considered moment have the same position as point *C* that belongs to slider.

The velocity of point B results from motion of the guide and it should be known. The guide may be in translational, rotational or plane motion.

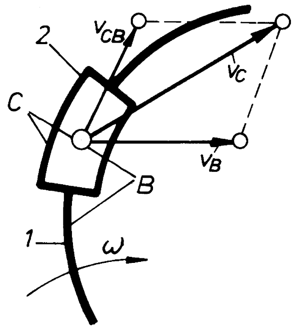


Fig. [Miller 1996]

Compound motion

Acceleration of point *C* is equal:

$$\vec{a}_C = \vec{a}_B + \vec{a}_{CB}$$

$\boldsymbol{a}_B$ – acceleration of point *B* in relations to the frame (stationary reference frame).

$\boldsymbol{a}_{CB}$ – acceleration of point *C* (that belongs to slider) relative to point *B* (that belongs to guide)

$$\vec{a}_{CB} = \vec{a}^{\,n}_{CB} + \vec{a}^{\,t}_{CB} + \vec{a}^{\,c}_{CB}$$

Normal acceleration | Tangential acceleration | Coriolis acceleration

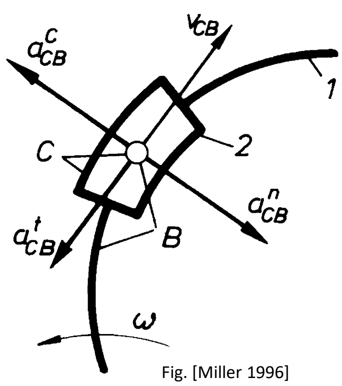


Fig. [Miller 1996]

Compound motion

a_{CB}^{n} Normal acceleration

$$a_{CB}^{n} = \frac{v_{CB}^{2}}{\rho}$$

The acceleration direction is consistent with the radius of the curvature of the guide and is directed towards its center

ρ – radius of the guide in point under consideration.

If the guide is rectilinear ($\rho = \infty$), normal acceleration is equal 0

$$\vec{a}_{CB}^{n} = \frac{v_{CB}^{2}}{\infty} = 0$$

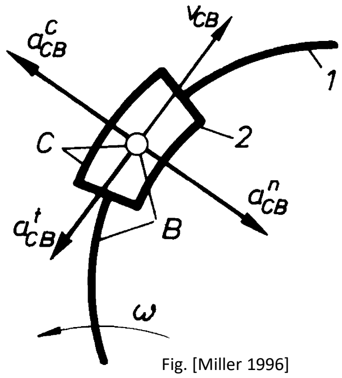


Fig. [Miller 1996]

Compound motion

a_{CB}^{t} Tangential acceleration

$$a_{CB}^{t} = \frac{dv_{CB}}{dt}$$

The direction of the tangential acceleration is tangent to the path (guide)

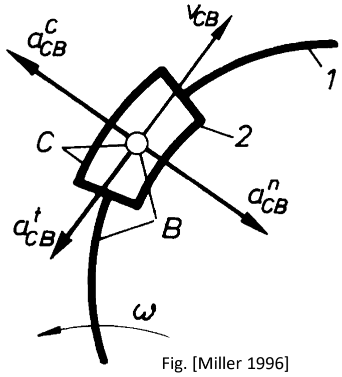


Fig. [Miller 1996]

Compound motion

a^{c}_{CB} Coriolis acceleration

$$a^{c}_{CB} = 2\omega \times v_{CB}$$

The direction of the Coriolis acceleration is determined by the rotation of the relative velocity vector $\boldsymbol{v}_{CB}$ through a right angle in the direction of angular velocity ω.

The Coriolis acceleration is 0 when the angular velocity ω = 0 (guide is not moving or makes translational motion) or the relative velocity v_{CB} = 0 (the slider is not moving with respect to the guide).

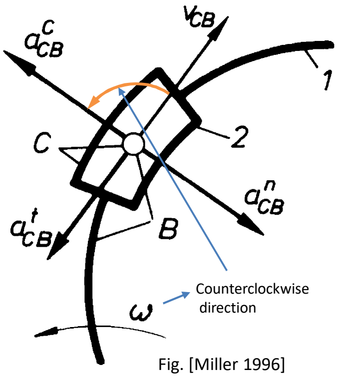


Fig. [Miller 1996]

Methods of kinematic analysis of mechanisms

Methods of analysis:
- analytical,
- numerical,
- graphical,
- experimental.

There are also their combinations, such as the combination of analytical and graphical methods – grapho-analytical methods.

Due to the form of classes and the advantages of simplicity of determination and help in understanding the essence of kinematic analysis, grapho-analytical methods will be used.

How many methods should an engineer know?

Analitycal methods

Based on the geometric dependencies, the motion of the mechanism can be described by mathematical formulas. For this purpose: vector notation, matrix notation, complex number, trigonometric or algebraic equations are used. In this way, dependencies describing the positions of the links are obtained. Then you can get formulas for velocity and acceleration by differentiating them with respect to time.

Analytical methods are quite difficult and laborious. However, the obtained results are valuable because:

- enable the analysis of the influence of the geometry of the mechanism on its kinematics,
- can be used for any mechanism of a certain type,
- a solution is obtained for all possible positions with very high accuracy.

Analytical methods - vector loop equation

It is one of the basic methods used to analyze plane mechanisms with analytical methods. Based on the kinematic diagram, a vector polygon is created. The beginnings and ends of vectors are determined by the positions of the kinematic pairs. Since the vectors form a polygon, the equation is:

$$\sum_{i=1}^{n} \vec{l}_i = \vec{l}_1 + \vec{l}_2 + \vec{l}_3 + \ldots + \vec{l}_n = 0$$

where $\boldsymbol{l}_n$ are vectors that belong to polygon

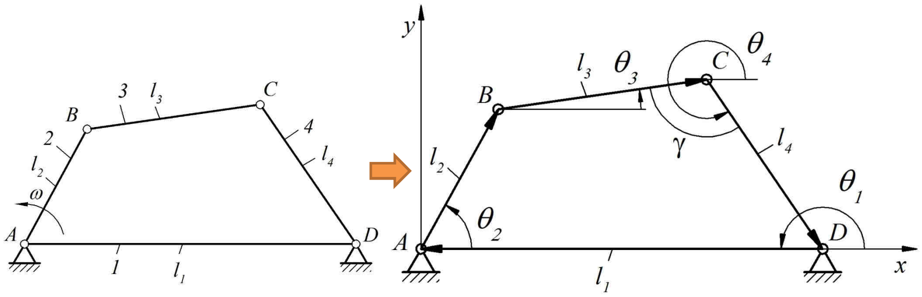
Analytical methods - vector loop equation (example)

Four bar mechanism

Two angles θ_3 and θ_4 as well as angular velocities and accelerations should be determined. The lengths of the links and the angles θ_1 and θ_2 are known.

The x-axis direction is collinear with the l_1 base. Angles are measured counterclockwise from the x-axis (the measuring direction can be either way, it is important to follow the assumptions throughout the whole calculation process).

Step 1: Draw and label the vectors



Analytical methods - vector loop equation (example)

Step 2: Write the equation $\sum_{i=1}^{n} \vec{l}_i = 0$

$$\vec{l}_1 + \vec{l}_2 + \vec{l}_3 + \vec{l}_4 = 0$$

And scalar equation: $\sum_{i=1}^{n} l_i \cos\theta_i = 0$ about *x axis* $\sum_{i=1}^{n} l_i sin\theta_i = 0$ and *y axis*

$$l_1\cos\theta_1 + l_2\cos\theta_2 + l_3\cos\theta_3 + l_4\cos\theta_4 = 0$$

$$l_2 sin\theta_2 + l_3 sin\theta_3 + l_4 sin\theta_4 = 0$$

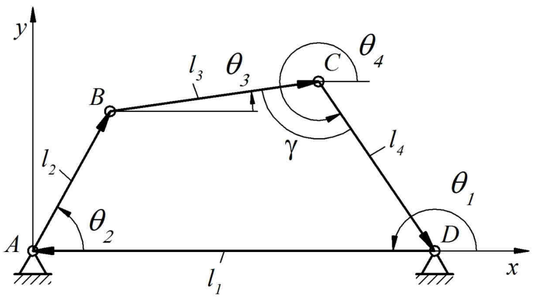
The final result of the solution are two equations [Myszka 2012]:

$$BD = \sqrt{L_1^2 + L_2^2 - 2L_1L_2\cos(\theta_2)}$$

$$\gamma = \arccos\left(\frac{L_3^2 + L_4^2 - BD^2}{2L_3L_4}\right)$$

$$\theta_3 = 2arctg\left(\frac{-L_2\sin\theta_2 + L_4\sin\gamma}{L_1 + L_3 - L_2\cos\theta_2 - L_4\cos\gamma}\right)$$

$$\theta_4 = 2arctg\left(\frac{L_2\sin\theta_2 - L_3\sin\gamma}{L_2\cos\theta_2 + L_4 - L_1 - L_3\cos\gamma}\right)$$



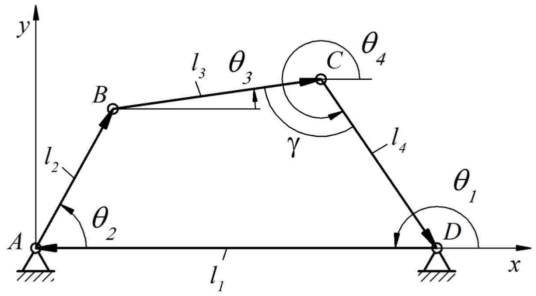
Analytical methods - vector loop equation (example)

In order to obtain the equation for angular velocity, first order derivative of angular displacement is done and second order derivative of angular displacement to get acceleration [Myszka 2012]:

$$\omega_3 = \omega_2\left[\frac{L_2\sin(\theta_4-\theta_2)}{L_3\sin\gamma}\right] \qquad \varepsilon_3 = \frac{\varepsilon_2 L_2\sin(\theta_2-\theta_4)+\omega_2^2 L_2\cos(\theta_2-\theta_4)-\omega_4^2 L_4+\omega_3^2 L_3\cos(\theta_4-\theta_3)}{L_3\sin(\theta_4-\theta_3)}$$

$$\omega_4 = \omega_2\left[\frac{L_2\sin(\theta_3-\theta_2)}{L_4\sin\gamma}\right] \qquad \varepsilon_4 = \frac{\varepsilon_2 L_2\sin(\theta_2-\theta_3)+\omega_2^2 L_2\cos(\theta_2-\theta_3)-\omega_3^2 L_3\cos(\theta_4-\theta_3)+\omega_3^2 L_3}{L_4\sin(\theta_4-\theta_3)}$$

The full derivation of the formulas for displacement, velocity and acceleration can be found in Młynarski 1992.



Numerical methods

Numerical methods have two main applications in the analysis of mechanisms:
1. Performing calculations in dedicated programs only on the basis of defining the mechanism and boundary conditions.

A significant group of dedicated programs uses the numerical method of multiboby simulation (MBS). It is possible to model planar (2D and 3D) and spatial mechanisms with rigid and deformable members, taking into account friction and contacts, damping, elasticity, external forces and moments.

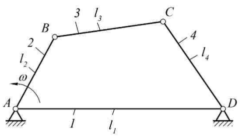
2. Obtaining results from equations for the full range of motion and various parameters of the mechanism. Also solving equations for that exact solution is not known and only this method is suitable.

Programming languages, spreadsheets and computing programs with a high-level programming language are used. It is possible to obtain results directly from algebraic equations without substituting specific data, through symbolic calculations (e.g. Mathematica, Matlab).

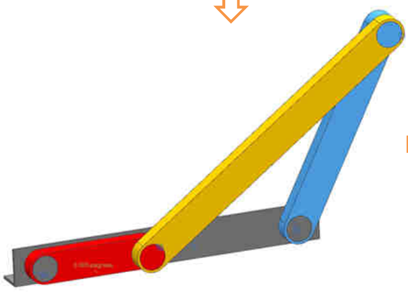
Numerical methods – example in NX

1. Performing calculations in dedicated programs only on the basis of defining the mechanism and boundary conditions.

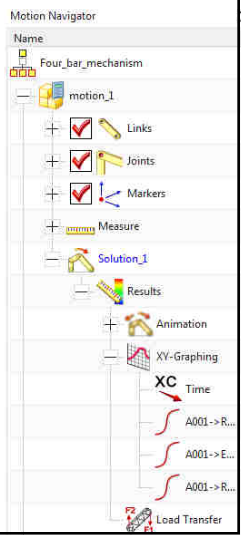
```
Determine the angular displacement, velocity and
acceleration for the rocker in four bar mechanism:

L1 = 0.5;  % Frame length [m]
L2 = 0.2;  % Crank length [m]
L3 = 0.6;  % Coupler length [m]
L4 = 0.4;  % Rocker length[m]
ω = 2Π     % Angular velocity[rad/s]
ε = 0      % angular acceleration[rad/s²]
```



CAD model



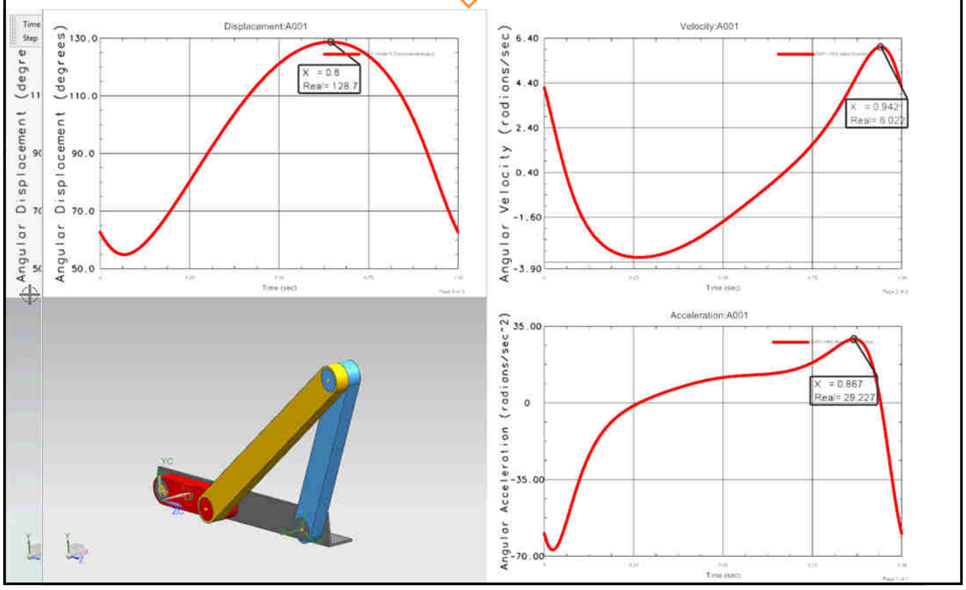
Mechanism definition



Numerical methods – example in NX

1. Performing calculations in dedicated programs only on the basis of defining the mechanism and boundary conditions.

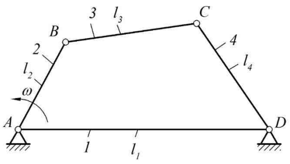
Results for rocker



Numerical methods – Matlab example

2. Obtaining results from equations for the full range of motion and various parameters of the mechanism.

Determine the angular displacement, velocity and acceleration for the rocker in four bar mechanism :



[Myszka 2012]:

$$\theta_4 = 2arctg\left(\frac{L_2 \sin\theta_2 - L_3 \sin\gamma}{L_2\cos\theta_2 + L_4 - L_1 - L_3\cos\gamma}\right)$$

$$\omega_4 = \omega_2\left[\frac{L_2\sin(\theta_3-\theta_2)}{L_4\sin\gamma}\right]$$

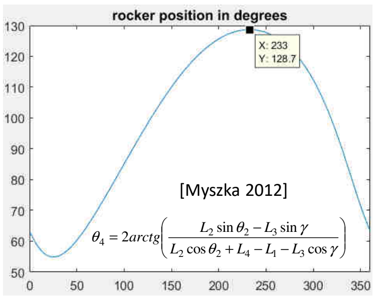
Equations in Matlab

$$\varepsilon_4 = -\frac{\varepsilon_2 L_2\sin(\theta_2-\theta_3)+\omega_2^2 L_2\cos(\theta_2-\theta_3)-\omega_4^2 L_4\cos(\theta_4-\theta_3)+\omega_3^2 L_3}{L_4\sin(\theta_4-\theta_3)}$$

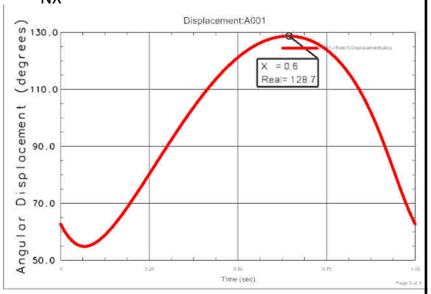
```
%% Calculating position, velocity and acceleration of rocker in
% four bar mechanism

L1 = 0.5;             % base lenght in [m]
L2 = 0.2;             % crank lenght in [m]
L3 = 0.6;             % coupler lenght in [m]
L4 = 0.4;             % rocker lenght in [m]
theta2 = (0:1:360)';  % angle between a base and crank

%% Calculating rocker position in four bar mechanism
% distance between revolute joint crank/coupler and revolute
% joint rocker/base

BD = sqrt(L1^2+L2^2-2*L1*L2*cosd(theta2));

% angle between coupler and rocker
gamma = acosd((L3^2+L4^2-BD.^2)/(2*L3*L4));

% rocker position
numerator_theta4 = L2*sind(theta2)-L3*sind(gamma);
denominator_theta4 = L2*cosd(theta2)+L4-L1-L3*cosd(gamma);
theta4 = 2*atand(numerator_theta4./denominator_theta4);
figure(1)
plot(theta4)
title('rocker position in degrees')
%% Calculating rocker velocity in four bar mechanism

omega2 = 2*pi;  % initial velocity of crank [rad/s]

% coupler position
numerator_theta3 = -L2*sind(theta2)+L4*sind(gamma);
denominator_theta3 = L1+L3-L2*cosd(theta2)-L4*cosd(gamma);
theta3 = 2*atand(numerator_theta3./denominator_theta3);
```

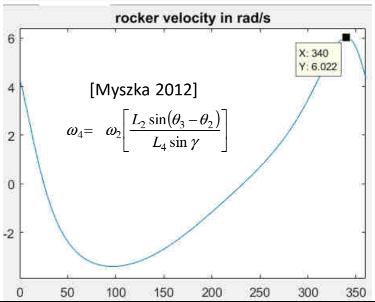
Numerical methods – Matlab example

2. Obtaining results from equations for the full range of motion and various parameters of the mechanism.



NX





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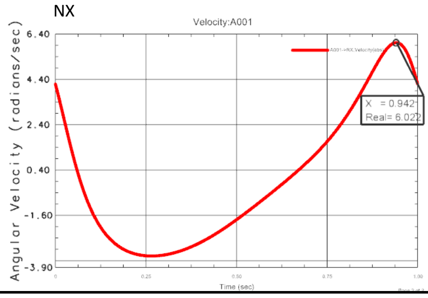
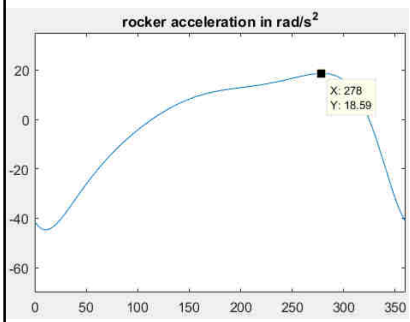


Numerical methods – Matlab example

2. Obtaining results from equations for the full range of motion and various parameters of the mechanism.

[Myszka 2012]

$$\varepsilon_4 = -\frac{\varepsilon_2 L_2 \sin(\theta_2-\theta_3)+\omega_2^2 L_2 \cos(\theta_2-\theta_3)-\omega_3^2 L_3 \cos(\theta_4-\theta_3)+\omega_3^2 L_3}{L_4 \sin(\theta_4-\theta_3)}$$



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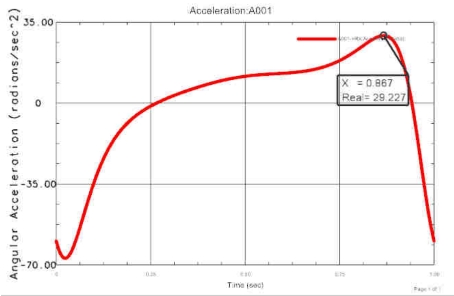
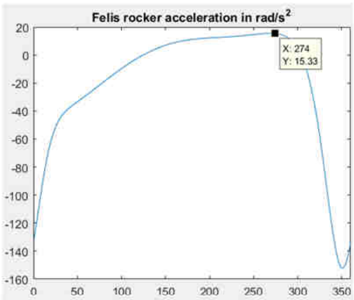
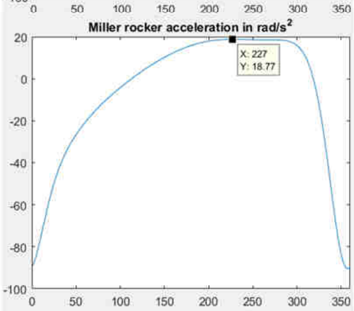


Numerical methods – Matlab example

2. Obtaining results from equations for the full range of motion and various parameters of the mechanism.





[Felis 2008]

$$\varepsilon_3 = -\frac{\omega_1^2 l_1 \cos(\varphi_1-\varphi_2)+\varepsilon_1 l_1 \sin(\varphi_1-\varphi_2)+\omega_2^2 l_2+\omega_3^2 l_3 \cos(\varphi_3-\varphi_2)}{l_3 \sin(\varphi_3-\varphi_2)}$$

Aby oznaczenia były zgodne z rysunkiem $\varepsilon_1 = \varepsilon_2$; $\varepsilon_2 = \varepsilon_3$; $\varepsilon_3 = \varepsilon_4$; $\omega_1 = \omega_2$ itd. $\varphi_1 = \theta_2$ itd.

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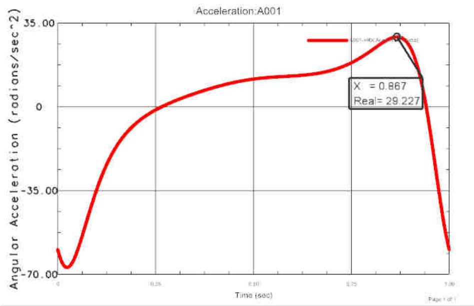


[Miller 1996]

$$\varepsilon_4 = \frac{\omega_2^2 l_2 \cos(\varphi_2-\varphi_3)+\omega_3^2 l_3 \cos(\varphi_4-\varphi_3)}{l_4 \sin(\varphi_4-\varphi_3)},$$

Numerical methods – Matlab example

2. Obtaining results from equations for the full range of motion and various parameters of the mechanism.

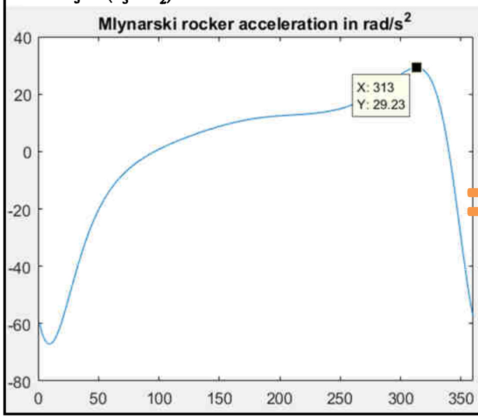
$$\varepsilon_2 = \omega_1^2 \left\{ \frac{l_1 l_3 \sin(\alpha_3-\alpha_2)\cos(\alpha_1-\alpha_2)[l_2\sin(\alpha_3-\alpha_2) - l_1\sin(\alpha_1-\alpha_3)]}{l_2\cdot l_3^2 \sin^3(\alpha_3-\alpha_2)} + \right.$$

[Młynarski 1992]

$$\left. - \frac{l_1^2 \sin(\alpha_1-\alpha_2)\cos(\alpha_3-\alpha_2)[l_2\sin(\alpha_1-\alpha_2)-l_3\sin(\alpha_1-\alpha_3)]}{l_2 l_3^2\sin^3(\alpha_3-\alpha_2)}\right\} +$$

$$+\frac{l_1\sin(\alpha_1-\alpha_2)}{l_3\sin(\alpha_3-\alpha_2)}\cdot\varepsilon_1$$

Variables must be substitute to be consistent with drawing
$\varepsilon_1 = \varepsilon_2$; $\varepsilon_2 = \varepsilon_4$; $\varphi_1 = \theta_2$; $\varphi_2 = \theta_3$; $\varphi_3 = \theta_4$



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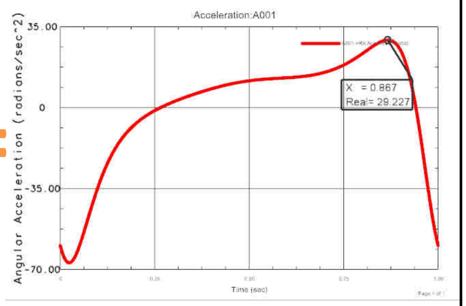


Grapho-analytical method

Method of determining velocity and accelerations by grapho-analytical method will be discussed. In order to determine the kinematic parameters, a kinematic diagram as well as velocity and acceleration diagrams should be drawn. It requires the adoption of appropriate drawing scale.
The drawing scale is defined as the ratio of the value of the physical quantity to the value of the drawing quantity:

$\kappa_l = \frac{l}{(l)}\left[\frac{m}{mm}\right]$ lenght scale,

$\kappa_v = \frac{v}{(v)}\left[\frac{m/s}{mm}\right]$ linear velocity scale,

$\kappa_a = \frac{a}{(a)}\left[\frac{m/s^2}{mm}\right]$ linear acceleration scale.

Example

1. Given: velocity v = 500 m/s, scale $\kappa_v = 10\left[\frac{m/s}{mm}\right]$. Find the length of the velocity vector in the drawing.

$$(v) = \frac{v}{\kappa_v}\left[\frac{m}{s}/\frac{m/s}{mm}\right] = \frac{500\ m/s}{10\frac{m/s}{mm}} = 50\ mm$$

2. Given: length of velocity (v) = 50 mm, scale $\kappa_v = 10\left[\frac{m/s}{mm}\right]$. Find the value of velocity.

$$v = (v)\kappa_v\left[mm\frac{m/s}{mm}\right] = 50\ mm\cdot 10\frac{\frac{m}{s}}{mm} = 500\ m/s$$

Grapho-analytical method

Velocity and acceleration diagrams will be discussed for:

1. Single three node link.
2. Assur group of II class.
2.1. Assur group of II class with revolute pair.
2.2. Assur group of II class with a sliding pair.
3. Assur group of III class with four links and revolute pairs.
4. An example of determining the velocity and acceleration for four bar mechanism.

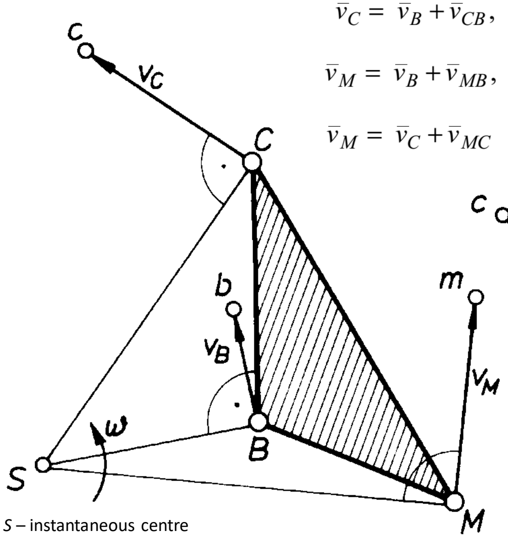
Grapho-analytical method

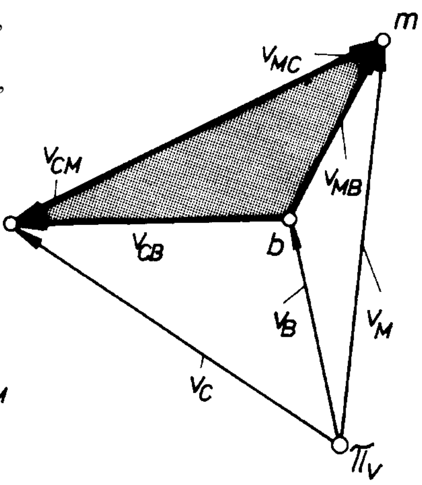
1. Velocity and acceleration diagrams for three node link

$$\bar{v}_C = \bar{v}_B + \bar{v}_{CB},$$

$$\bar{v}_M = \bar{v}_B + \bar{v}_{MB},$$

$$\bar{v}_M = \bar{v}_C + \bar{v}_{MC}$$





S – instantaneous centre

Fig. [Miller 1996]

Π_v – Velocity origin - start point for absolute velocity vectors

Grapho-analytical method

1. Velocity and acceleration diagrams for three node link

ΔBCM ~ Δbcm

The triangle Δ BCM is similar to the triangle Δbcm and rotated by an angle of 90 ° according to the angular velocity ω (it is counterclockwise in the figure). This property can be used to check the correctness of the velocity diagram or to determine a third velocity.

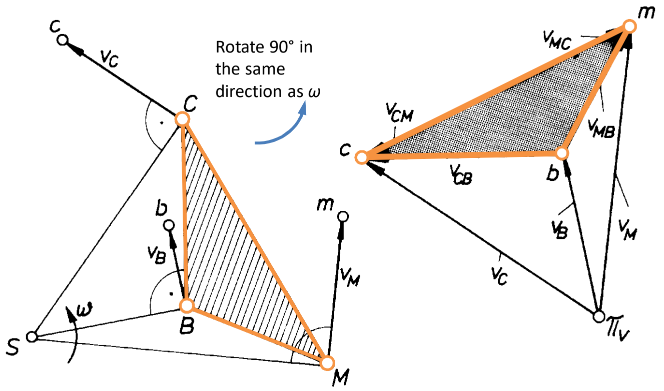
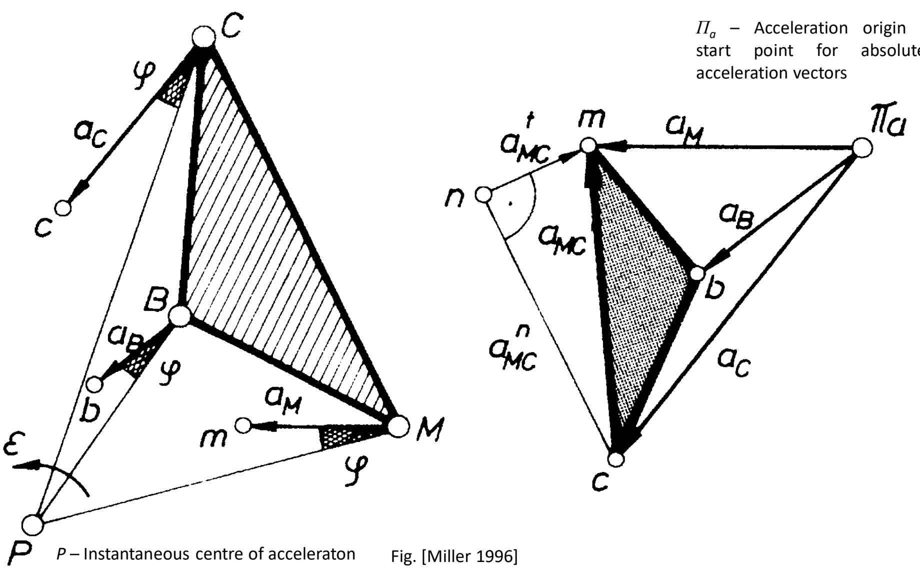


Fig. [Miller 1996]

Grapho-analytical method

1. Velocity and acceleration diagrams for three node link

Π_a – Acceleration origin - start point for absolute acceleration vectors



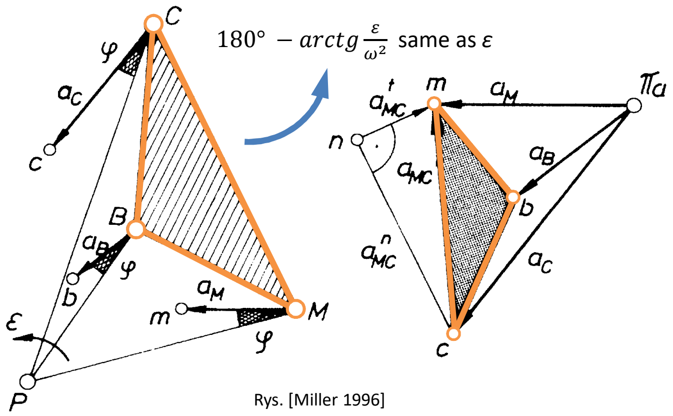
P – Instantaneous centre of acceleraton

Fig. [Miller 1996]

Grapho-analytical method

1. Velocity and acceleration diagrams for three node link

ΔBCM ~ Δbcm
The triangle ΔBCM is similar to the triangle Δbcm and rotated through the angle $180° - arctg\frac{\varepsilon}{\omega^2}$ according to the angular acceleration ε (in the figure it is counterclockwise). This property can be used to check the correctness of the acceleration diagram or to determine the acceleration of third point.



Rys. [Miller 1996]

Grapho-analytical method

2.1. ***Veolcity and acceleration diagrams for Assur group of II class with revolute pair***

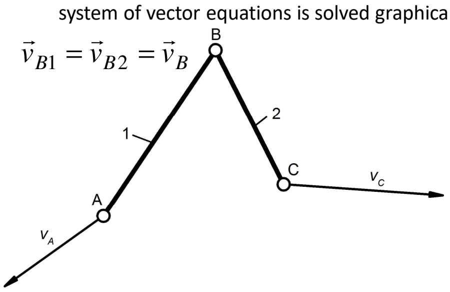
The velocities and accelerations of points *A* and *C* are known. Determine the velocity and acceleration of point *B* using the grapho-analytical method.

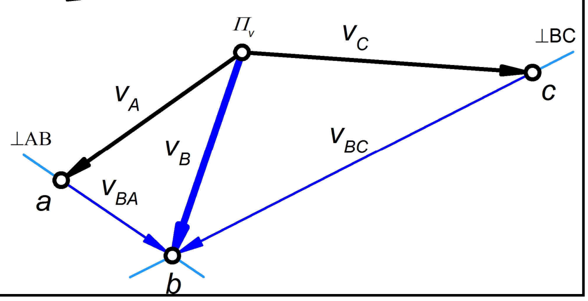
In order to determine the velocity of point *B*, the system of vector equations is solved graphically:

$$\begin{cases} \vec{v}_B = \underline{\underline{\vec{v}_A}} + \underline{\vec{v}_{BA}} \\ \quad\quad\quad\quad\quad \perp AB \\ \vec{v}_B = \underline{\underline{\vec{v}_C}} + \underline{\vec{v}_{BC}} \\ \quad\quad\quad\quad\quad \perp BC \end{cases}$$

One underline indicates the direction is known, two underlines: values and direction

$\vec{v}_{B1} = \vec{v}_{B2} = \vec{v}_B$





Grapho-analytical method

2.1. *Veolcity and acceleration diagrams for Assur group of II class with revolute pair*

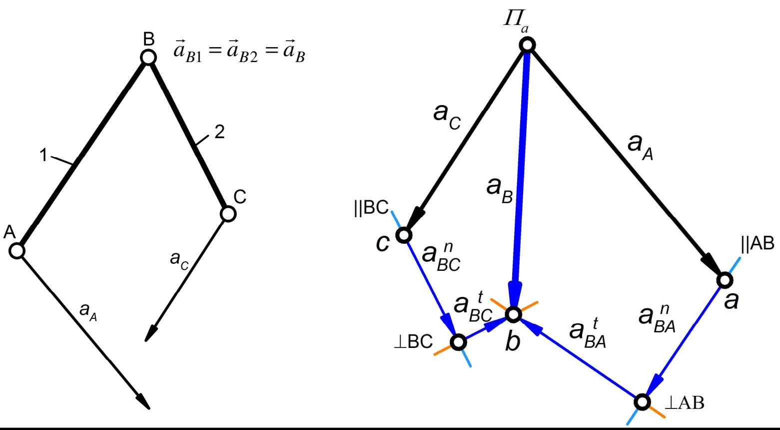
In order to determine the acceleration of point *B*, the system of vector equations is graphically and analytically solved:

$$\begin{cases} \vec{a}_B = \vec{a}_A + \vec{a}_{BA} = \underline{\underline{\vec{a}_A}} + \underset{\parallel AB}{\underline{\underline{\vec{a}^n_{BA}}}} + \underset{\perp AB}{\underline{\vec{a}^t_{BA}}} \\ \vec{a}_B = \vec{a}_C + \vec{a}_{BC} = \underline{\underline{\vec{a}_C}} + \underset{\parallel BC}{\underline{\underline{\vec{a}^n_{BC}}}} + \underset{\perp BC}{\underline{\vec{a}^t_{BC}}} \end{cases}$$

The value of the relative normal acceleration is analytically determined:

$$a^n_{BA} = \frac{v^2_{BA}}{|AB|}$$

$$a^n_{BC} = \frac{v^2_{BC}}{|BC|}$$



Grapho-analytical method

2.2. *Veolcity and acceleration diagrams for Assur group of II class with sliding pair*

The velocities and accelerations of points *A* and *C* are known. Determine the velocities and accelerations of points B_1 and B_2 using the grapho-analytical method (Fig. 1).

In this case, there is too little infromation to directly determine the velocity of points B_1 and B_2. It is required to know the velocity of two points for one element. For this purpose, two additional points are determined: C_1 belonging to the link 1 and A_2 belonging to the link 2 (it can be imagined as welding plates to the elements on which the introduced points are located - Fig. 2). The position of these points coincides in the considered position with points A1 and C2. Taking such a position of points simplifies the determination of the directions of relative velocities and the drawing of velocity diagram.

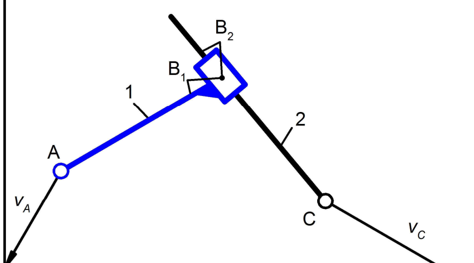


Fig. 1

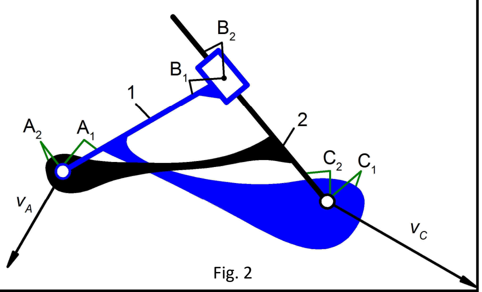


Fig. 2

Grapho-analytical method

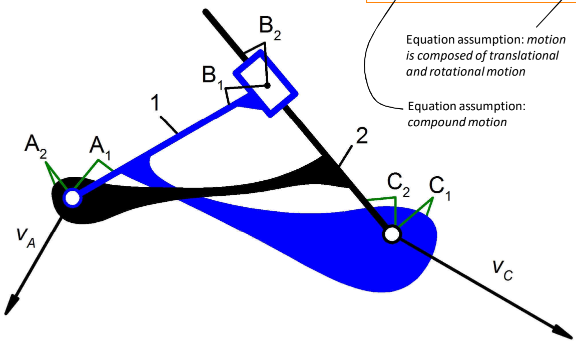
2.2. *Veolcity and acceleration diagrams for Assur group of II class with sliding pair*

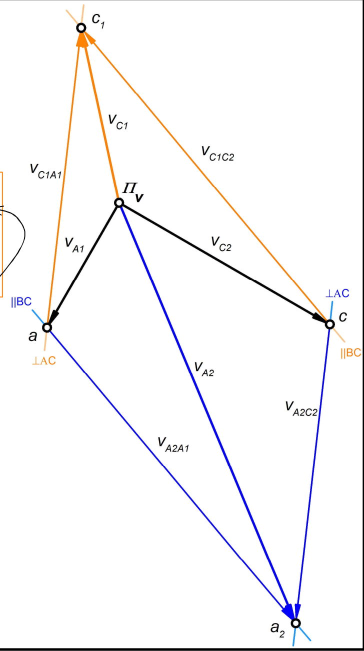
Now it is necessary to determine the velocities of the assumed points, for which systems of vector equations can be written, for the point A_2:

$$\begin{cases} \vec{v}_{A2} = \underline{\underline{\vec{v}_{C2}}} + \underset{\perp AC}{\vec{v}_{A2C2}} \\ \vec{v}_{A2} = \underline{\underline{\vec{v}_{A1}}} + \underset{\parallel BC}{\vec{v}_{A2A1}} \end{cases}$$

and for the point C_1:

$$\begin{cases} \vec{v}_{C1} = \underline{\underline{\vec{v}_{A1}}} + \underset{\perp AC}{\vec{v}_{C1A1}} \\ \vec{v}_{C1} = \underline{\underline{\vec{v}_{C2}}} + \underset{\parallel BC}{\vec{v}_{C1C2}} \end{cases}$$





Grapho-analytical method

2.2. *Veolcity and acceleration diagrams for Assur group of II class with sliding pair*

Knowing the velocities of two different points for each of the link, the determination of the velocity of the points B_1 and B_2 may have the following course:

I. Treating the plane motion of links as composed of translational and rotational motion, one can write systems of vector equations and solve them graphically:

$$\begin{cases} \vec{v}_{B1} = \underline{\underline{\vec{v}_{A1}}} + \underset{\perp AB}{\vec{v}_{B1A1}} \\ \vec{v}_{B1} = \underline{\underline{\vec{v}_{C1}}} + \underset{\perp BC}{\vec{v}_{B1C1}} \end{cases} \qquad \begin{cases} \vec{v}_{B2} = \underline{\underline{\vec{v}_{A2}}} + \underset{\perp AB}{\vec{v}_{B2A2}} \\ \vec{v}_{B2} = \underline{\underline{\vec{v}_{C2}}} + \underset{\perp BC}{\vec{v}_{B2C1}} \end{cases}$$

II. Determine the instantaneous centres and directions of the unknown velocities, and then:

a) Calculate angular velocity (e.g. $\omega = v_{A1}/|O_1A|$) and unknown velocitiec $v_{B1} = \omega \cdot |O_1B|$ i $v_{B2} = \omega \cdot |O_2B|$. The angular velocity ω is the same for both links and it results from the construction of the structural group (rigid connection in the link 1 of the rectilinear part with the slider).

b) Using the velocity projection method, graphically determine or calculate the velocities of points from the relationship: $v_{B1} = \dfrac{v_{A1}\cos(\beta_{A1})}{\cos(\alpha_{B1})}$; $v_{B2} = \dfrac{v_{C2}\cos(\beta_{C2})}{\cos(\alpha_{B2})}$

III. Determine the velocity vectors using the graphical method from the similarity of figures (in this case triangles).

Methods I and III will be presented

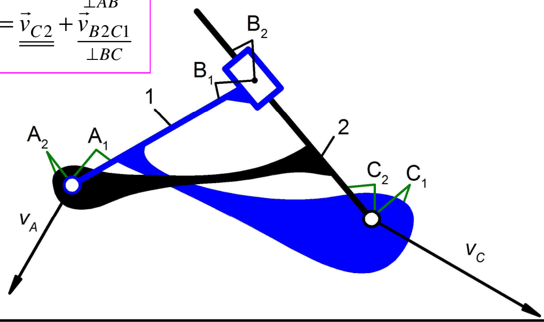
Grapho-analytical method

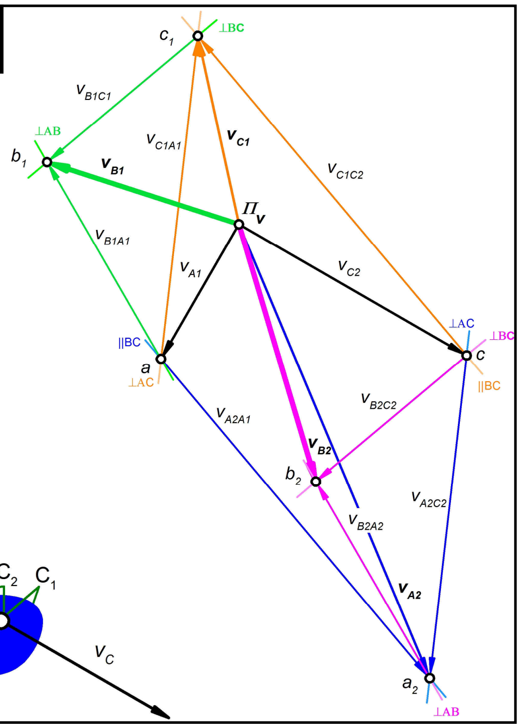
2.2. *Veolcity and acceleration diagrams for Assur group of II class with sliding pair*

Now it is necessary to determine the velocity of the points B_1 and B_2 for which systems of vector equations can be written:

$$\begin{cases} \vec{v}_{B1} = \underline{\underline{\vec{v}_{A1}}} + \underset{\perp AB}{\underline{\vec{v}_{B1A1}}} \\ \vec{v}_{B1} = \underline{\underline{\vec{v}_{C1}}} + \underset{\perp BC}{\underline{\vec{v}_{B1C1}}} \end{cases}$$

$$\begin{cases} \vec{v}_{B2} = \underline{\underline{\vec{v}_{A2}}} + \underset{\perp AB}{\underline{\vec{v}_{B2A2}}} \\ \vec{v}_{B2} = \underline{\underline{\vec{v}_{C2}}} + \underset{\perp BC}{\underline{\vec{v}_{B2C1}}} \end{cases}$$





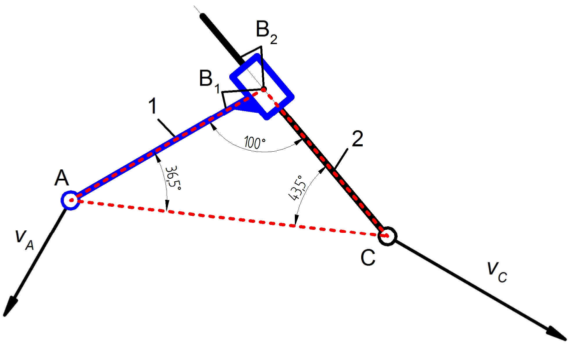
Grapho-analytical method

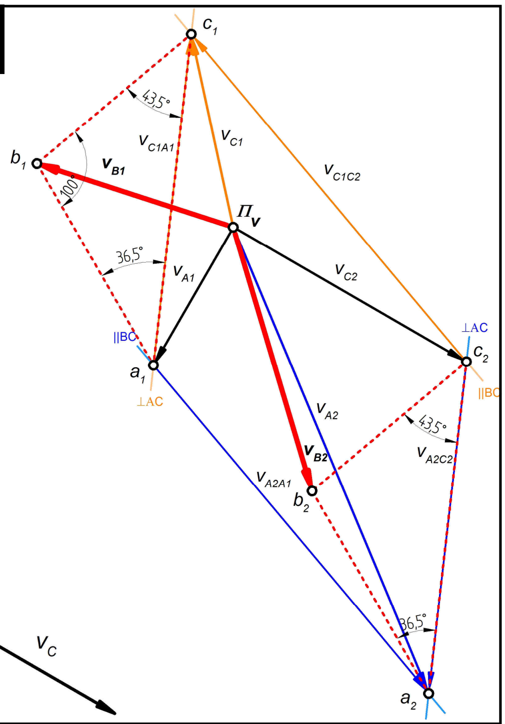
2.2. *Veolcity and acceleration diagrams for Assur group of II class with sliding pair*

Velocities of the B_1 and B_2 points determined from the similarity of triangles:

$\Delta A_1B_1C_1 \sim \Delta a_1b_1c_1$

$\Delta A_2B_2C_2 \sim \Delta a_2b_2c_2$





Grapho-analytical method

2.2. ***Veolcity and acceleration diagrams for Assur group of II class with sliding pair***

The procedure of determining accelerations is very similar to determining the velocities. The accelerations for two additional points C_1 and A_2 should also be determined from the equation systems:

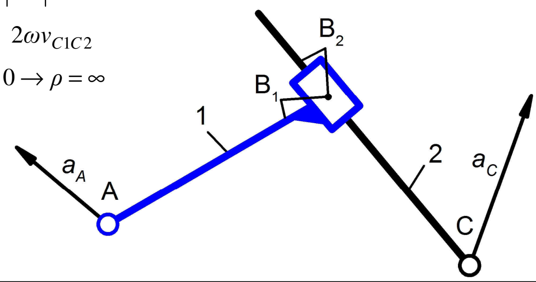
$$\begin{cases} \vec{a}_{A2} = \underline{\underline{\vec{a}_{C2}}} + \underset{\parallel AC}{\underline{\underline{\vec{a}^{\,n}_{A2C2}}}} + \underset{\perp AC}{\underline{\vec{a}^{\,t}_{A2C2}}} \\ \vec{a}_{A2} = \underline{\underline{\vec{a}_{A1}}} + \underset{\perp BC}{\underline{\underline{\vec{a}^{\,c}_{A2A1}}}} + \underset{\parallel BC}{\underline{\vec{a}^{\,t}_{A2A1}}} \end{cases}$$

$$\begin{cases} \vec{a}_{C1} = \underline{\underline{\vec{a}_{A1}}} + \underset{\parallel AC}{\underline{\underline{\vec{a}^{\,n}_{C1A1}}}} + \underset{\perp AC}{\underline{\vec{a}^{\,t}_{C1A1}}} \\ \vec{a}_{C1} = \underline{\underline{\vec{a}_{C2}}} + \underset{\perp BC}{\underline{\underline{\vec{a}^{\,c}_{C1C2}}}} + \underset{\parallel BC}{\underline{\vec{a}^{\,t}_{C1C2}}} \end{cases}$$

$$a^n_{A2C2} = \frac{v^2_{A2C2}}{|AC|} \qquad a^n_{C1A1} = \frac{v^2_{C1A1}}{|AC|}$$

$$a^c_{A2A1} = 2\omega v_{A2A1} \qquad a^c_{C1C2} = 2\omega v_{C1C2}$$

$$\vec{a}^{\,n}_{A2A1} = 0 \rightarrow \rho = \infty \qquad \vec{a}^{\,n}_{C1C2} = 0 \rightarrow \rho = \infty$$

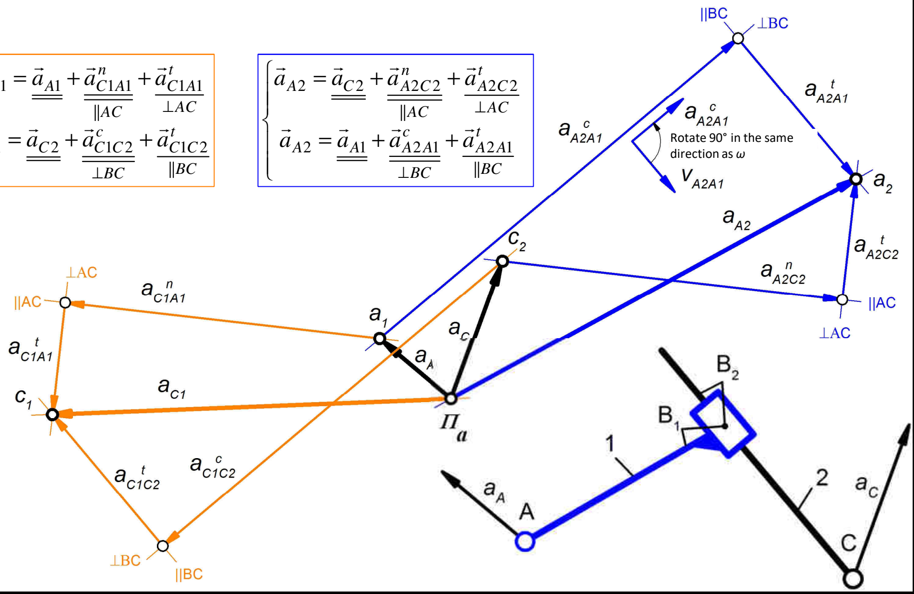


Grapho-analytical method

2.2. ***Veolcity and acceleration diagrams for Assur group of II class with sliding pair***

$$\begin{cases} \vec{a}_{C1} = \underline{\underline{\vec{a}_{A1}}} + \underset{\parallel AC}{\underline{\underline{\vec{a}^{\,n}_{C1A1}}}} + \underset{\perp AC}{\underline{\vec{a}^{\,t}_{C1A1}}} \\ \vec{a}_{C1} = \underline{\underline{\vec{a}_{C2}}} + \underset{\perp BC}{\underline{\underline{\vec{a}^{\,c}_{C1C2}}}} + \underset{\parallel BC}{\underline{\vec{a}^{\,t}_{C1C2}}} \end{cases}$$

$$\begin{cases} \vec{a}_{A2} = \underline{\underline{\vec{a}_{C2}}} + \underset{\parallel AC}{\underline{\underline{\vec{a}^{\,n}_{A2C2}}}} + \underset{\perp AC}{\underline{\vec{a}^{\,t}_{A2C2}}} \\ \vec{a}_{A2} = \underline{\underline{\vec{a}_{A1}}} + \underset{\perp BC}{\underline{\underline{\vec{a}^{\,c}_{A2A1}}}} + \underset{\parallel BC}{\underline{\vec{a}^{\,t}_{A2A1}}} \end{cases}$$



Grapho-analytical method

2.2. ***Veolcity and acceleration diagrams for Assur group of II class with sliding pair***

Knowing the accelerations of two different points for each of the link, the determination of the accelerations of the points B_1 and B_2 may have the following course:

I. Treating the plane motion of links as composed of translational and rotational motion, one can write systems of vector equations and solve them graphically:

$$\begin{cases} \vec{a}_{B1} = \underline{\underline{\vec{a}_{A1}}} + \underset{\parallel AB}{\underline{\underline{\vec{a}^{n}_{B1A1}}}} + \underset{\perp AB}{\underline{\vec{a}^{t}_{B1A1}}} \\ \vec{a}_{B1} = \underline{\underline{\vec{a}_{C1}}} + \underset{\parallel BC}{\underline{\underline{\vec{a}^{n}_{B1C1}}}} + \underset{\perp BC}{\underline{\vec{a}^{t}_{B1C1}}} \end{cases} \qquad \begin{cases} \vec{a}_{B2} = \underline{\underline{\vec{a}_{A2}}} + \underset{\parallel AB}{\underline{\underline{\vec{a}^{n}_{B2A2}}}} + \underset{\perp AB}{\underline{\vec{a}^{t}_{B2A2}}} \\ \vec{a}_{B2} = \underline{\underline{\vec{a}_{C2}}} + \underset{\parallel BC}{\underline{\underline{\vec{a}^{n}_{B2C2}}}} + \underset{\perp BC}{\underline{\vec{a}^{t}_{B2C2}}} \end{cases}$$

II. Determine the instantaneous centres of acceleration and then unknown accelerations. The angular acceleration ε is identical for both members and it results from the construction of the structural group (a rigid connection in the member 1 of the rectilinear part with the slider).

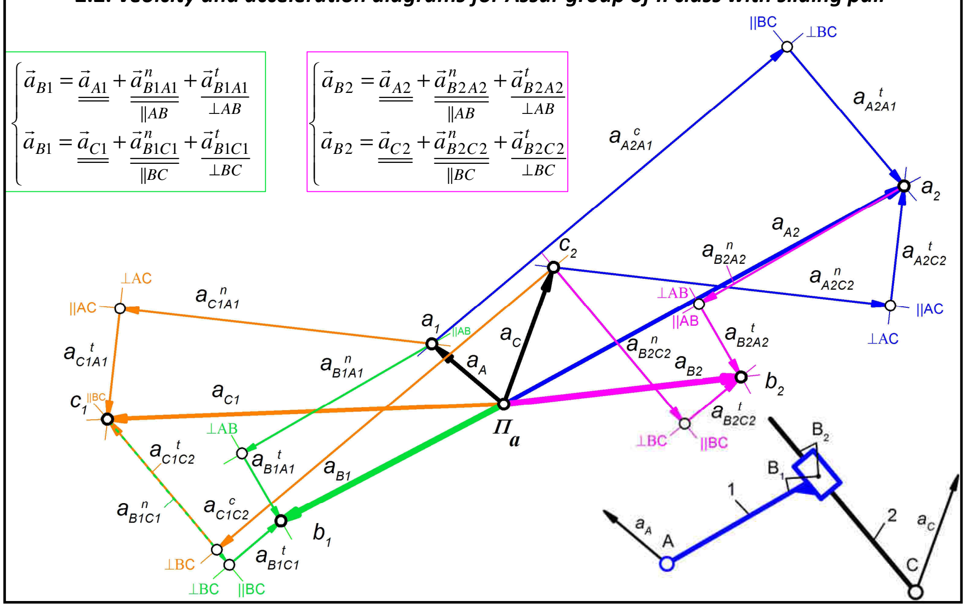
III. Determine the acceleration vectors using the graphical method from the similarity of figures (in this case triangles).

Method I will be presented

Grapho-analytical method

2.2. ***Veolcity and acceleration diagrams for Assur group of II class with sliding pair***

$$\begin{cases} \vec{a}_{B1} = \underline{\underline{\vec{a}_{A1}}} + \underset{\parallel AB}{\underline{\underline{\vec{a}^{n}_{B1A1}}}} + \underset{\perp AB}{\underline{\vec{a}^{t}_{B1A1}}} \\ \vec{a}_{B1} = \underline{\underline{\vec{a}_{C1}}} + \underset{\parallel BC}{\underline{\underline{\vec{a}^{n}_{B1C1}}}} + \underset{\perp BC}{\underline{\vec{a}^{t}_{B1C1}}} \end{cases} \qquad \begin{cases} \vec{a}_{B2} = \underline{\underline{\vec{a}_{A2}}} + \underset{\parallel AB}{\underline{\underline{\vec{a}^{n}_{B2A2}}}} + \underset{\perp AB}{\underline{\vec{a}^{t}_{B2A2}}} \\ \vec{a}_{B2} = \underline{\underline{\vec{a}_{C2}}} + \underset{\parallel BC}{\underline{\underline{\vec{a}^{n}_{B2C2}}}} + \underset{\perp BC}{\underline{\vec{a}^{t}_{B2C2}}} \end{cases}$$



Grapho-analytical method

3. *Veolcity and acceleration diagrams for Assur group of III class*

The velocities and accelerations of points *D*, *E* and *F* of the class III Assur group shown in Figure 1 are known. The velocities and accelerations of points *A*, *B* and *C* shall be determined.

For this purpose, the so-called Assur's points *R*, *S* or *T* lying at the intersection of two straight lines passing through pairs of two-node links (Fig. 2) are introduced. They are rigidly connected to the three-node ABC link. One Assur's point is enough to find unknown velocities and accelerations.

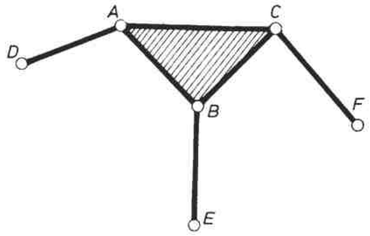


Fig. 1. Class III Assur group
[based on Miller 1996]

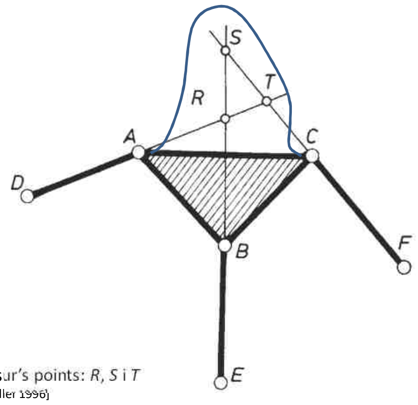


Fig. 2. Assur's points: *R*, *S* i *T*
[based on Miller 1996]

Grapho-analytical method

3. *Veolcity and acceleration diagrams for Assur group of III class*

The velocity of point *R* will be determined first, which will allow in the next step to calculate the velocity of point *A*, *B* or *C*. Treating the motion of the ABCR link as comopsed of translational and rotarional motion, the system of equations is obtained:

$$\begin{cases} \vec{v}_R = \vec{v}_A + \underset{\perp AR}{\underline{\vec{v}_{RA}}} \\ \vec{v}_R = \vec{v}_B + \underset{\perp BR}{\underline{\vec{v}_{RB}}} \end{cases}$$

There are too many unknowns to get a solution. Points *A* and *B* also belong to the links AD and BE for which the velocities of points D and E are known, so there are relationships:

$$\vec{v}_A = \underline{\underline{\vec{v}_D}} + \underset{\perp AD}{\underline{\vec{v}_{AD}}}; \qquad \vec{v}_B = \underline{\underline{\vec{v}_E}} + \underset{\perp BE}{\underline{\vec{v}_{BE}}}$$

Substituting the equations into the system of equations we get:

$$\begin{cases} \vec{v}_R = \underline{\underline{\vec{v}_D}} + \underset{\perp AD}{\underline{\vec{v}_{AD}}} + \underset{\perp AR}{\underline{\vec{v}_{RA}}} \\ \vec{v}_R = \underline{\underline{\vec{v}_E}} + \underset{\perp BE}{\underline{\vec{v}_{BE}}} + \underset{\perp BR}{\underline{\vec{v}_{RB}}} \end{cases}$$

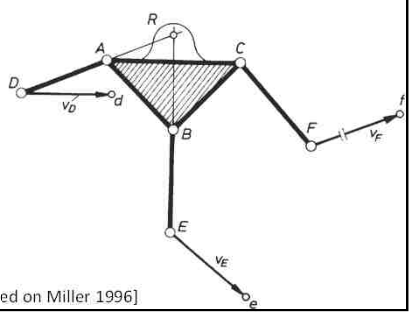


Fig. [Based on Miller 1996]

Grapho-analytical method

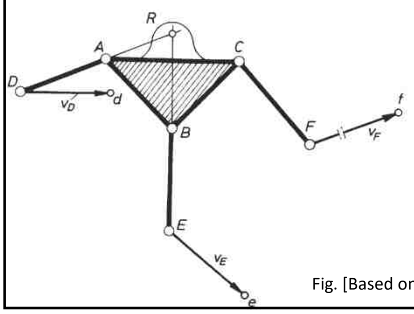
3. *Veolcity and acceleration diagrams for Assur group of III class*

The reduction of the directions of relative velocities to two is due to the non-accidental selection of the *R* point, and it is already possible to solve the system of equations:

$$\begin{cases} \vec{v}_R = \underline{\underline{\vec{v}_D}} + \underset{\perp AD}{\underline{\vec{v}_{AD}}} + \underset{\perp AR}{\underline{\vec{v}_{RA}}} = \underline{\underline{\vec{v}_D}} + \underset{\perp AD}{\underline{\vec{v}_{RD}}} \\ \vec{v}_R = \underline{\underline{\vec{v}_E}} + \underset{\perp BE}{\underline{\vec{v}_{BE}}} + \underset{\perp BR}{\underline{\vec{v}_{RB}}} = \underline{\underline{\vec{v}_E}} + \underset{\perp BE}{\underline{\vec{v}_{RE}}} \end{cases}$$

The velocity of any point of the three-node link can be determined, for point *C* the system of equations has the form

$$\begin{cases} \vec{v}_C = \underline{\underline{\vec{v}_F}} + \underset{\perp CF}{\underline{\vec{v}_{CF}}} \\ \vec{v}_C = \underline{\underline{\vec{v}_R}} + \underset{\perp CR}{\underline{\vec{v}_{CR}}} \end{cases}$$



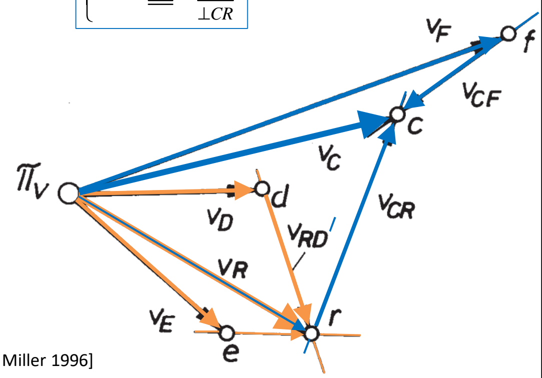


Fig. [Based on Miller 1996]

Grapho-analytical method

3. *Veolcity and acceleration diagrams for Assur group of III class*

Determining the accelerations also begins with determining the acceleration of the *R* point, the system of equations has the form:

$$\begin{cases} \vec{a}_R = \vec{a}_A + \underset{\parallel AR}{\underline{\underline{\vec{a}^n_{RA}}}} + \underset{\perp AR}{\underline{\vec{a}^t_{RA}}} \\ \vec{a}_R = \vec{a}_B + \underset{\parallel BR}{\underline{\underline{\vec{a}^n_{RB}}}} + \underset{\perp BR}{\underline{\vec{a}^t_{RB}}} \end{cases}$$

since

$$\begin{cases} \vec{a}_A = \underline{\underline{\vec{a}_D}} + \underset{\parallel AD}{\underline{\underline{\vec{a}^n_{AD}}}} + \underset{\perp AD}{\underline{\vec{a}^t_{AD}}} \\ \vec{a}_B = \underline{\underline{\vec{a}_E}} + \underset{\parallel BE}{\underline{\underline{\vec{a}^n_{BE}}}} + \underset{\perp BE}{\underline{\vec{a}^t_{BE}}} \end{cases}$$

so

$$\begin{cases} \vec{a}_R = \underline{\underline{\vec{a}_D}} + \underset{\parallel AD}{\underline{\underline{\vec{a}^n_{AD}}}} + \underset{\perp AD}{\underline{\vec{a}^t_{AD}}} + \underset{\parallel AR}{\underline{\underline{\vec{a}^n_{RA}}}} + \underset{\perp AR}{\underline{\vec{a}^t_{RA}}} \\ \vec{a}_R = \underline{\underline{\vec{a}_E}} + \underset{\parallel BE}{\underline{\underline{\vec{a}^n_{BE}}}} + \underset{\perp BE}{\underline{\vec{a}^t_{BE}}} + \underset{\parallel BR}{\underline{\underline{\vec{a}^n_{RB}}}} + \underset{\perp BR}{\underline{\vec{a}^t_{RB}}} \end{cases}$$

Where:

$$\vec{a}^n_{AD} = \frac{v^2_{AD}}{|AD|};\quad \vec{a}^n_{RA} = \frac{v^2_{RA}}{|RA|};\quad \vec{a}^n_{BE} = \frac{v^2_{BE}}{|BE|};\quad \vec{a}^n_{RB} = \frac{v^2_{RB}}{|RB|}$$

The acceleration of any point of the three-node link can already be determined, for point *C* the system of equations has the form:

$$\begin{cases} \vec{a}_C = \vec{a}_R + \underset{\parallel CR}{\underline{\underline{\vec{a}^n_{CR}}}} + \underset{\perp CR}{\underline{\vec{a}^t_{CR}}} \\ \vec{a}_C = \vec{a}_F + \underset{\parallel CF}{\underline{\underline{\vec{a}^n_{CF}}}} + \underset{\perp CF}{\underline{\vec{a}^t_{CF}}} \end{cases}$$

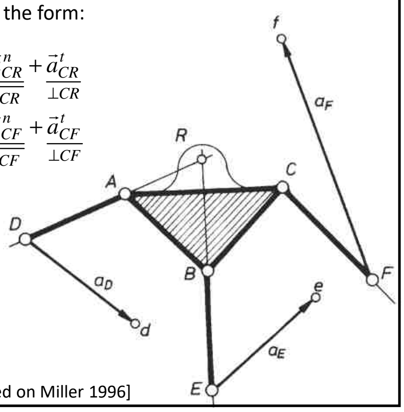


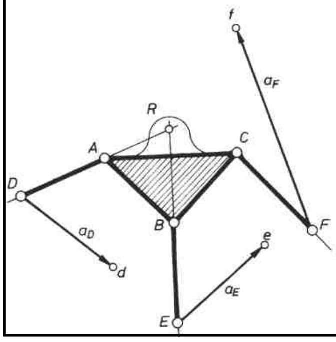
Fig. [Based on Miller 1996]

Grapho-analytical method

3. *Veolcity and acceleration diagrams for Assur group of III class*

$$\begin{cases} \vec{a}_R = \underline{\underline{\vec{a}_D}} + \underset{\|AD}{\underline{\underline{\vec{a}^n_{AD}}}} + \underset{\perp AD}{\vec{a}^t_{AD}} + \underset{\|AR}{\underline{\underline{\vec{a}^n_{RA}}}} + \underset{\perp AR}{\vec{a}^t_{RA}} \\ \vec{a}_R = \underline{\underline{\vec{a}_E}} + \underset{\|BE}{\underline{\underline{\vec{a}^n_{BE}}}} + \underset{\perp BE}{\vec{a}^t_{BE}} + \underset{\|BR}{\underline{\underline{\vec{a}^n_{RB}}}} + \underset{\perp BR}{\vec{a}^t_{RB}} \end{cases}$$

$$\begin{cases} \vec{a}_C = \vec{a}_R + \underset{\|CR}{\underline{\underline{\vec{a}^n_{CR}}}} + \underset{\perp CR}{\vec{a}^t_{CR}} \\ \vec{a}_C = \vec{a}_F + \underset{\|CF}{\underline{\underline{\vec{a}^n_{CF}}}} + \underset{\perp CF}{\vec{a}^t_{CF}} \end{cases}$$



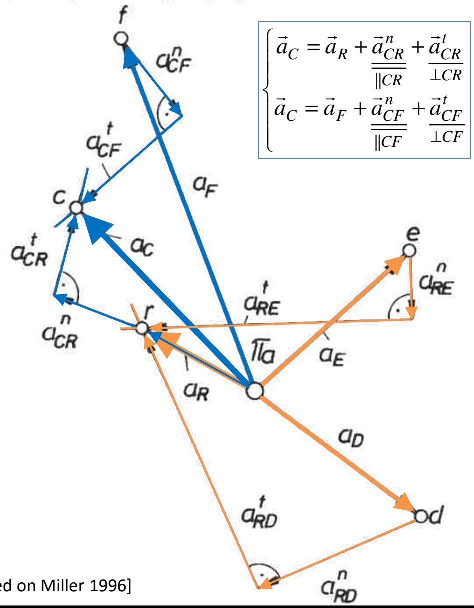


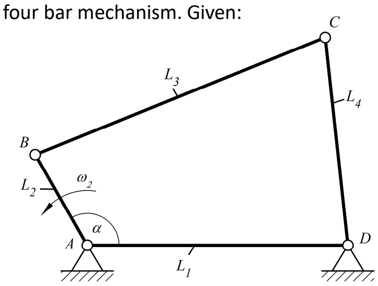
Fig. [Based on Miller 1996]

Grapho-analytical method

4. An example of determining the velocity and acceleration for four bar mechanism

Determine the velocity and acceleration for rocker of four bar mechanism. Given:

```
L1 = 0.5;  % Frame length [m]
L2 = 0.2;  % Crank length [m]
L3 = 0.6;  % Coupler length [m]
L4 = 0.4;  % Rocker length[m]
ω2 = 2Π     % Angular velocity[rad/s]
ε2 = 0      % angular acceleration[rad/s²]
α = 120     % angular position of crank[°]
```



Velocity of point $B = B_2 = B_3$ is equal:

$$v_B = \omega_1 l_{AB} = 2\pi \cdot 0{,}2 = 0{,}4\pi \ \text{ m/s}$$

The velocity scale must be determined for drawing velocity diagram.

$$\kappa_v = 0{,}01 \left[\frac{\text{m}}{\text{s}} \frac{1}{\text{mm}}\right]$$

Length of velocity vector v_B

$$(v_B) = \frac{v_B}{\kappa_v} = \frac{0{,}4\pi}{0{,}01} = 40\pi \ \text{ mm}$$

Grapho-analytical method

4. An example of determining the velocity and acceleration for four bar mechanism

The velocity of point C is given by the equation:

$$\underset{\perp CD}{\underline{\vec{v}_C}} = \underset{\perp AB}{\underline{\underline{\vec{v}_B}}} + \underset{\perp BC}{\underline{\vec{v}_{CB}}}$$

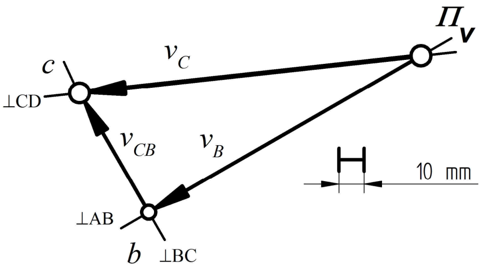
The length of vectors v_C and v_{CB} were read from velocity diagram and velocities were calculated:

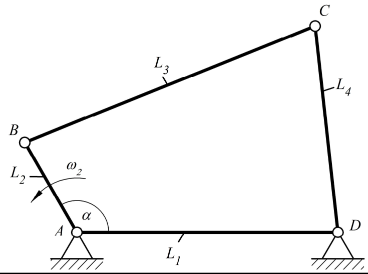
$$v_C = (v_C)\kappa_v = 137{,}2902018 \cdot 0{,}01 = 1{,}372902018 \ \frac{\text{m}}{\text{s}}$$

$$v_{CB} = (v_{CB})\kappa_v = 55{,}292246 \cdot 0{,}01 = 0{,}55292246 \ \frac{\text{m}}{\text{s}}$$

The angular velocity is:

$$\omega_4 = \frac{v_C}{|CD|} = \frac{1{,}372902018}{0{,}4} = 3{,}432255045 \ \frac{\text{rad}}{\text{s}}$$





Grapho-analytical method

4. An example of determining the velocity and acceleration for four bar mechanism

The acceleration of point C is:

$$\vec{a}_C = \vec{a}_B + \vec{a}_{CB}$$

and after substituting the component accelerations:

$$\underset{\parallel CD}{\underline{\underline{\vec{a}_C^n}}} + \underset{\perp CD}{\underline{\vec{a}_C^t}} = \underset{\parallel AB}{\underline{\underline{\vec{a}_B}}} + \underset{\parallel BC}{\underline{\underline{\vec{a}_{CB}^n}}} + \underset{\perp BC}{\underline{\vec{a}_{CB}^t}}$$

The known normal accelerations should be calculated, the acceleration scale is assumed $\kappa_a = 0{,}05 \left[\frac{\text{m}}{\text{s}^2}\frac{1}{\text{mm}}\right]$

$$a_C^n = \frac{v_C^2}{|CD|} = \frac{1{,}372902018^2}{0{,}4} = 4{,}71214987757 \ \frac{\text{m}}{\text{s}^2}$$

$$(a_C^n) = \frac{a_C^n}{\kappa_a} = \frac{4{,}71214987757}{0{,}05} = 94{,}24299755142 \ \text{mm}$$

$$a_B = \frac{v_B^2}{|AB|} = \frac{(0{,}4\pi)^2}{0{,}2} = 7{,}89568352087 \ \frac{\text{m}}{\text{s}^2}$$

$$(a_B) = \frac{a_B}{\kappa_a} = \frac{7{,}89568352087}{0{,}05} = 157{,}91367041742 \ \text{mm}$$

$$a_{CB}^n = \frac{v_{CB}^2}{|BC|} = \frac{0{,}55292246^2}{0{,}6} = 0{,}50953874462 \ \frac{\text{m}}{\text{s}^2}$$

$$(a_{CB}^n) = \frac{a_{CB}^n}{\kappa_a} = \frac{0{,}50953874462}{0{,}05} = 10{,}19077489241 \ \text{mm}$$

Grapho-analytical method

4. An example of determining the velocity and acceleration for four bar mechanism

$$\underset{\parallel CD}{\underline{\underline{\vec{a}_C^n}}} + \underset{\perp CD}{\underline{\vec{a}_C^t}} = \underset{\parallel AB}{\underline{\underline{\vec{a}_B}}} + \underset{\parallel BC}{\underline{\underline{\vec{a}_{CB}^n}}} + \underset{\perp BC}{\underline{\vec{a}_{CB}^t}}$$

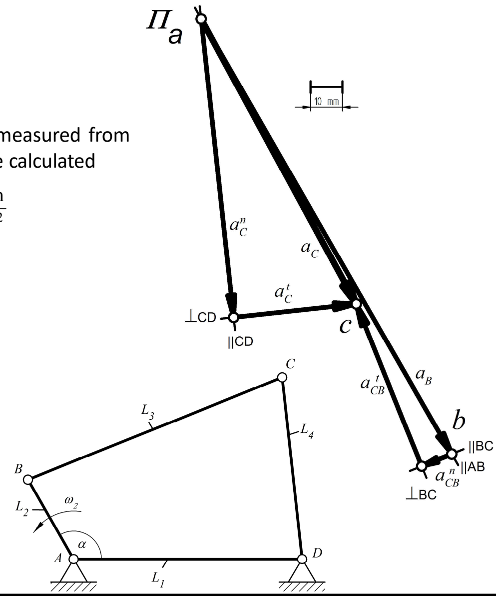
The lengths of the vectors a_C and a_C^t were measured from the diagram and the linear accelerations were calculated

$$a_C = (a_C)\kappa_a = 101{,}9405928 \cdot 0{,}05 = 5{,}09702964 \ \frac{\mathrm{m}}{\mathrm{s}^2}$$

$$a_C^t = (a_C^t)\kappa_a = 38{,}8605438 \cdot 0{,}05 = 1{,}94302719 \ \frac{\mathrm{m}}{\mathrm{s}^2}$$

and angular acceleration

$$\varepsilon_4 = \frac{a_C^t}{|CD|} = \frac{1{,}94302719}{0{,}4} = 4{,}857567975 \ \frac{\mathrm{rad}}{\mathrm{s}^2}$$



Grapho-analytical method

4. An example of determining the velocity and acceleration for four bar mechanism

Comparison of results for the analytical, numerical and grapho-analytical methods

Results for the grapho-analytical method (velocity and acceleration diagrams were made in the SolidEdge program):

ω_4 = **3,43**2255045 rad/s
ε_4 = **4,8**57567975 rad/s^2

Results for analytical formulas (calculations carried out in Matlab):

ω_4 = **3.24409266**7733456 rad/s
ε_4 = **4.44415340**7551584 rad/s^2

Results for the numerical method (mechanism simulation and results from the NX program):

ω_4 = **3,24409259**368118 rad/s
ε_4 = **4,44415324**003924 rad/s^2

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