

$$\int (2x+1)^3 dx = \left. \begin{array}{l} 2x+1=t \\ 2dx=dt \end{array} \right\} = \frac{1}{2} \int t^3 dt = \quad 16.26.$$

$$= \frac{1}{2} \cdot \frac{1}{4} t^4 + c = \frac{1}{8} (2x+1)^4 + c$$

$$\int \frac{1}{(3x-2)^4} dx = \left. \begin{array}{l} 3x-2=t \\ 3dx=dt \\ dx=1/3 dt \end{array} \right\} = \frac{1}{3} \int t^{-4} dt = \quad 16.27.$$

$$= \frac{1}{3} \cdot \frac{t^{-3}}{-3} + c = -\frac{1}{9} \frac{1}{(3x-2)^3} + c$$

$$\int \frac{3x-4}{x^2-x-6} dx = (*) \quad 16.28.$$

$$\Delta = 1+24=25, \quad x_1 = \frac{1-5}{2} = -2, \quad x_2 = 3$$

$$\frac{3x-4}{x^2-x-6} = \frac{A}{x+2} + \frac{B}{x-3}$$

$$3x-4 = A(x-3) + B(x+2)$$

$$x=3: \quad 9-4 = 0 + 5B \Rightarrow B=1$$

$$x=-2: \quad -6-4 = -5A + 0 \Rightarrow A=2$$

$$(*) = 2 \int \frac{1}{x+2} dx + \int \frac{1}{x-3} dx =$$

$$= 2 \ln|x+2| + \ln|x-3| + c$$

$$\int \frac{2x-3}{x^2-3x+3} dx = \ln|x^2-3x+3| + c \quad 16.29.$$

$$\int \frac{6x-13}{x^2-\frac{7}{2}x+\frac{3}{2}} dx = (*)$$

16.32.

$$\Delta = \frac{49}{4} - 6 = \frac{49}{4} - \frac{24}{4} = \frac{25}{4}, \quad x_1 = \frac{\frac{7}{2} - \frac{5}{2}}{2} = \frac{1}{2}, \quad x_2 = 3$$

$$\frac{6x-13}{x^2-\frac{7}{2}x+\frac{3}{2}} = \frac{A}{x-\frac{1}{2}} + \frac{B}{x-3}$$

$$6x-13 = A(x-3) + B(x-\frac{1}{2})$$

$$x=3 \Rightarrow 5 = \frac{5}{2}B \Rightarrow B=2$$

$$x=\frac{1}{2} \Rightarrow -10 = -\frac{5}{2}A \Rightarrow A=4$$

$$\begin{aligned} (*) &= 4 \int \frac{1}{x-\frac{1}{2}} dx + 2 \int \frac{1}{x-3} dx = \\ &= 4 \ln|x-\frac{1}{2}| + 2 \ln|x-3| + C \end{aligned}$$

$$\int \frac{4x-5}{2x^2-5x+3} dx = \ln|2x^2-5x+3| + C$$

16.33.

$$\int \frac{5x+11}{x^2+3x-10} dx = (*)$$

16.34.

$$\Delta = 9+40=49, \quad x_1 = \frac{-3-7}{2} = -5, \quad x_2 = 2$$

$$\frac{5x+11}{x^2+3x-10} = \frac{A}{x+5} + \frac{B}{x-2}$$

$$5x+11 = A(x-2) + B(x+5)$$

$$x=2: \quad 21 = 7B \Rightarrow B=3$$

$$x=-5: \quad -14 = -4A \Rightarrow A=2$$

$$\begin{aligned} (*) &= 2 \int \frac{1}{x+5} dx + 3 \int \frac{1}{x-2} dx = \\ &= 2 \ln|x+5| + 3 \ln|x-2| + C \end{aligned}$$

$$\int \frac{1}{6x^2 - 13x + 6} dx = (*)$$

16.37.

$$\Delta = 169 - 144 = 25, \quad x_1 = \frac{13-5}{12} = \frac{8}{12} = \frac{2}{3}, \quad x_2 = \frac{13+5}{12} = \frac{18}{12} = \frac{3}{2}$$

$$\begin{aligned} \frac{1}{6x^2 - 13x + 6} &= \frac{1}{6(x - \frac{2}{3})(x - \frac{3}{2})} = \frac{1}{(3x-2)(2x-3)} = \\ &= \frac{A}{3x-2} + \frac{B}{2x-3} \end{aligned}$$

$$1 = A(2x-3) + B(3x-2)$$

$$x = \frac{2}{3} : 1 = -\frac{5}{3}A \Rightarrow A = -\frac{3}{5}$$

$$x = \frac{3}{2} : 1 = \frac{5}{2}B \Rightarrow B = \frac{2}{5}$$

$$\begin{aligned} (*) &= -\frac{3}{5} \int \frac{1}{3x-2} dx + \frac{2}{5} \int \frac{1}{2x-3} dx = \\ &= -\frac{1}{5} \ln |3x-2| + \frac{1}{5} \ln |2x-3| + C = \\ &= \frac{1}{5} \ln \left| \frac{2x-3}{3x-2} \right| + C \end{aligned}$$

$$\int \frac{5+x}{10x+x^2} dx = \frac{1}{2} \int \frac{10+2x}{10x+x^2} dx = \frac{1}{2} \ln |10x+x^2| + C \quad 16.38.$$

$$\int \frac{7x}{4+5x^2} dx = \frac{7}{10} \int \frac{10x}{4+5x^2} dx = \frac{7}{10} \ln |4+5x^2| + C \quad 16.39.$$

16.44.

$$\int \frac{2x-1}{x^2-6x+8} dx = \int \frac{2x-1}{(x-3)^2} dx = (*)$$

$$\Delta = 36 - 36 = 0, \quad x_0 = \frac{-b}{2a} = \frac{6}{2} = 3$$

$$\frac{2x-1}{(x-3)^2} = \frac{A}{(x-3)^2} + \frac{B}{x-3}$$

$$2x-1 = A + B(x-3)$$

$$2x-1 = A - 3B + Bx$$

$$\begin{cases} B = 2 \\ A - 3B = -1 \end{cases} \Rightarrow A - 6 = -1 \Rightarrow A = 5$$

$$\begin{aligned} (*) &= 5 \int \frac{1}{(x-3)^2} dx + 2 \int \frac{1}{x-3} dx = \\ &= -5 \frac{1}{x-3} + 2 \ln|x-3| + C \end{aligned}$$

16.45.

$$\int \frac{x-1}{4x^2-4x+1} dx = \int \frac{x-1}{(2x-1)^2} dx = (*)$$

$$\frac{x-1}{(2x-1)^2} = \frac{A}{(2x-1)^2} + \frac{B}{2x-1}$$

$$x-1 = A + B(2x-1)$$

$$x-1 = A - B + 2Bx$$

$$\begin{cases} 2B = 1 \\ A - B = -1 \end{cases} \Rightarrow \begin{cases} B = \frac{1}{2} \\ A = -\frac{1}{2} \end{cases}$$

$$(*) = -\frac{1}{2} \int \frac{1}{(2x-1)^2} dx + \frac{1}{2} \int \frac{1}{2x-1} dx =$$

$$= \left\{ \begin{array}{l} 2x-1 = t \\ 2dx = dt \end{array} \right\} = -\frac{1}{2} \int t^{-2} dt + \frac{1}{4} \ln|2x-1| + C =$$

$$= +\frac{1}{4} t^{-1} + \frac{1}{4} \ln|2x-1| + C = \frac{1}{4} \cdot \frac{1}{(2x-1)} + \frac{1}{4} \ln|2x-1| + C.$$

$$\int \frac{2x-1}{x^2-2x+5} dx = \begin{cases} \Delta = 4-20 = -16, (x^2-2x+5)' = 2x-2 & 16.54. \\ 2x-1 = (2x-2) + 1 \\ x^2-2x+5 = (x-1)^2 + 4 \end{cases}$$

$$= \int \frac{2x-2+1}{x^2-2x+5} dx = \int \frac{2x-2}{x^2-2x+5} dx + \int \frac{1}{x^2-2x+5} dx =$$

$$= \ln |x^2-2x+5| + \int \frac{1}{(x-1)^2+4} dx =$$

$$= \ln |x^2-2x+5| + \frac{1}{4} \int \frac{1}{\left(\frac{x-1}{2}\right)^2+1} dx = \begin{cases} \frac{x-1}{2} = t \\ \frac{1}{2} dx = dt \end{cases}$$

$$= \ln |x^2-2x+5| + \frac{1}{4} \cdot 2 \int \frac{1}{t^2+1} dt =$$

$$= \ln |x^2-2x+5| + \frac{1}{2} \operatorname{arctg} t + C = \ln |x^2-2x+5| + \frac{1}{2} \operatorname{arctg} \left(\frac{x-1}{2}\right) + C$$

$$\int \frac{2x-10}{x^2-2x+10} dx = \begin{cases} \Delta = 4-40 = -36, (x^2-2x+10)' = 2x-2 & 16.55. \\ 2x-10 = (2x-2) + 8 \\ x^2-2x+10 = (x-1)^2+9 \end{cases}$$

$$= \int \frac{2x-2-8}{x^2-2x+10} dx = \int \frac{2x-2}{x^2-2x+10} dx - 8 \int \frac{1}{x^2-2x+10} dx$$

$$= \ln |x^2-2x+10| - 8 \int \frac{1}{(x-1)^2+9} dx =$$

$$= \ln |x^2-2x+10| - 8 \cdot \frac{1}{3} \int \frac{1}{\left(\frac{x-1}{3}\right)^2+1} dx = \begin{cases} \frac{x-1}{3} = t \\ \frac{1}{3} dx = dt \end{cases}$$

$$= \ln |x^2-2x+10| - \frac{8}{3} \int \frac{1}{t^2+1} dt =$$

$$= \ln |x^2-2x+10| - \frac{8}{3} \operatorname{arctg} t + C =$$

$$= \ln |x^2-2x+10| - \frac{8}{3} \operatorname{arctg} \left(\frac{x-1}{3}\right) + C.$$

16.56.

$$\int \frac{2x-20}{x^2-8x+25} dx = \begin{cases} \Delta = 64-100 = -36, & (x^2-8x+25)' = 2x-8 \\ 2x-20 = (2x-8) - 12 \\ x^2-8x+25 = (x-4)^2 + 9 \end{cases}$$

$$= \int \frac{2x-8-12}{x^2-8x+25} dx = \int \frac{2x-8}{x^2-8x+25} dx - 12 \int \frac{1}{x^2-8x+25} dx$$

$$= \ln|x^2-8x+25| - 12 \int \frac{1}{(x-4)^2+9} dx = \begin{cases} x-4=3t \\ dx=3dt \end{cases}$$

$$= \ln|x^2-8x+25| - 12 \int \frac{1}{9t^2+9} \cdot 3dt = \begin{cases} t = \frac{x-4}{3} \end{cases}$$

$$= \ln|x^2-8x+25| - 12 \cdot \frac{3}{9} \int \frac{1}{t^2+1} dt =$$

$$= \ln|x^2-8x+25| - 4 \arctg t + c =$$

$$= \ln|x^2-8x+25| - 4 \arctg \left( \frac{x-4}{3} \right) + c$$